Transformation of Rigid Structures

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joint work with
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Motivations

- Stochastic graph transformation good for modelling *adaptive networks*, where topology and state changes are interdependent
  - biological systems
  - social or technical networks
- Need a *constructive approach* towards model creation, refinement and analysis following example of *kappa*

Goals

- Make it easy to define domain-specific languages, including constraints for
  - patterns and state graphs
  - rules and matches
- Axiomatise conditions for constructive approach
- Be able to *engineer* approaches to satisfy such conditions

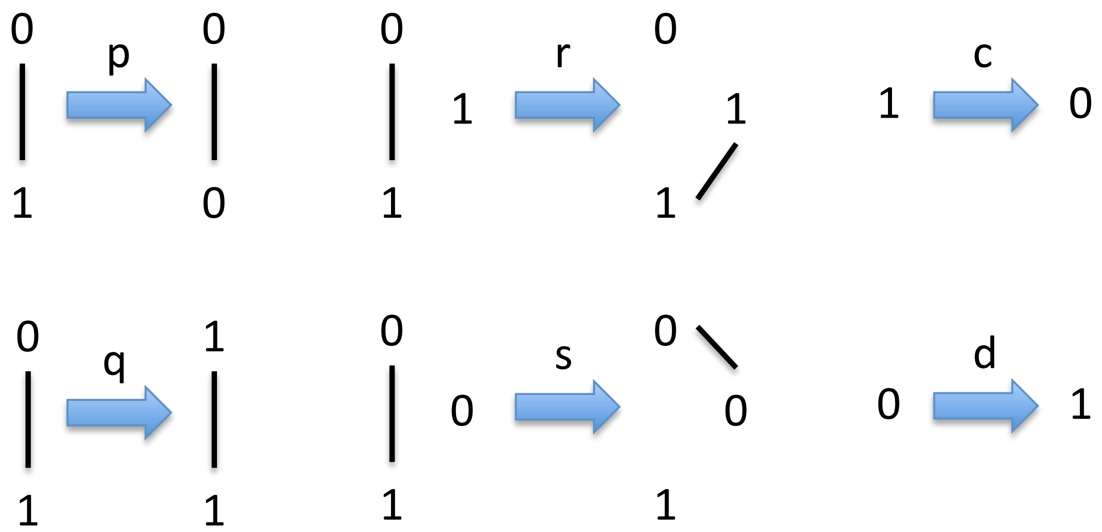
Social Network Model

Individuals hold one of two opinions (votes 0 or 1), coevolving with their connections:

- If two connected individuals hold different opinions,
  - one is converted to the opinion of the other, or
  - their link is broken and one makes a new connection to a random individual of the same opinion
- Individuals can change opinions spontaneously

[Graph Fission in Evolving Voter Model, Durret et al., 2012]

Intuitively



- p, q: be converted
- r, s: split up and reconnect
- c, d: change spontaneously

Questions

How does the number of occurrences of such patterns change

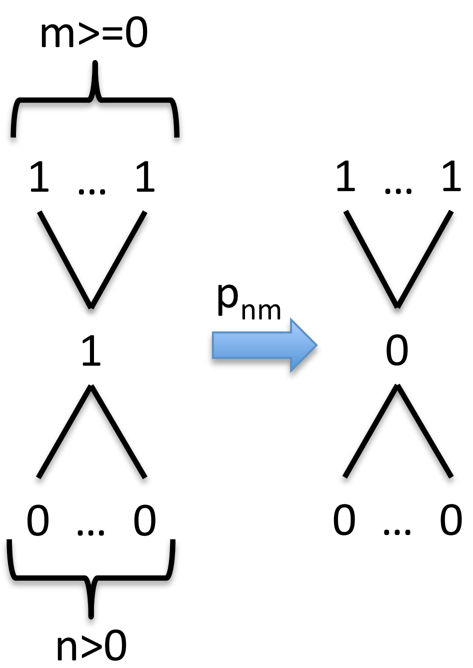
- with the application of individual rules?
- over time?

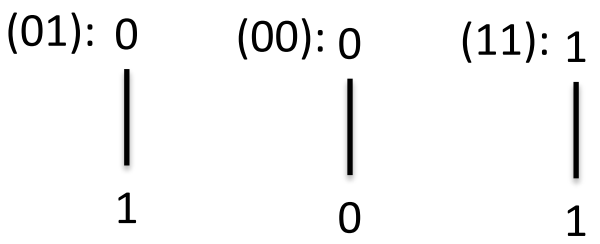
Given observed number of occurrences of patterns such as

- 0 or 1 nodes
- 01, 11, or 00 links

how to consistently assign rates to rules?

How do p, q, r, s change numbers of occurrences of patterns?



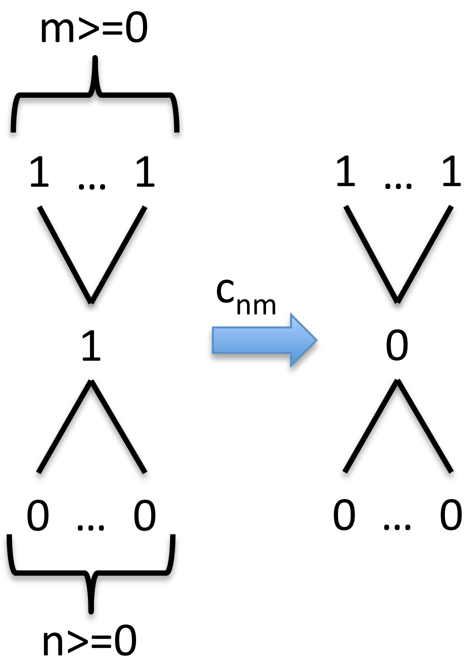


- p_{nm} for n+m <= max degree
  - creates m-n occurrences of (01)
  - creates 2n occurrences of (00)
  - destroys 2m occurrences of (11)

- dually for q_{nm}

- r, s each destroy one (01) and create two (00), (11), resp.

How do c, d change numbers of occurrences of patterns?



- c_{nm} for n+m <= max degree
  - creates m-n occurrences of (01)
  - creates 2n occurrences of (00)
  - destroys 2m occurrences of (11)

- dually for d_{nm}

Depends on the context? But can we produce a refined set of rules, where these numbers are fixed?

Petrification

Produce balanced model: each rule creates or destroys fixed number of occurrences

- How to prevent application of an instance rule to larger context?
- →Constraints on matches preventing certain contexts: *open maps*

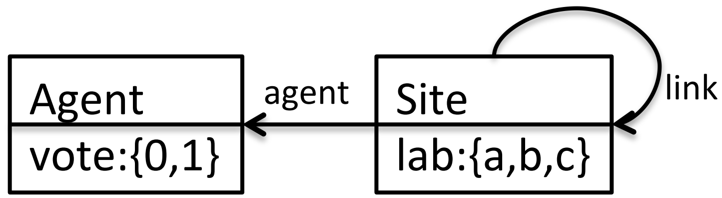
Preserve stochastic behaviour: (functional) stochastic bisimilation between original and refined model

- How to guarantee 1-to-1 between rule/match pairs of original and refined models?
- →Constraints on graphs to guarantee unique extension of match to added context: *rigidity*

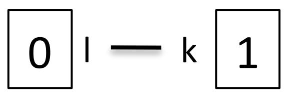
Methodology

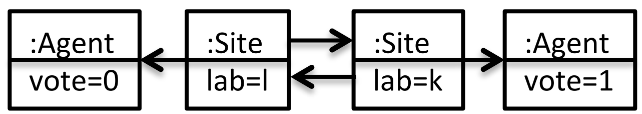
1. Define explicit model: type graph and rules
2. Define constraints: patterns and states
3. Select minimal permissible rules
4. Derive constraints for matches
5. Petrify

1. Explicit Model: Type Graph

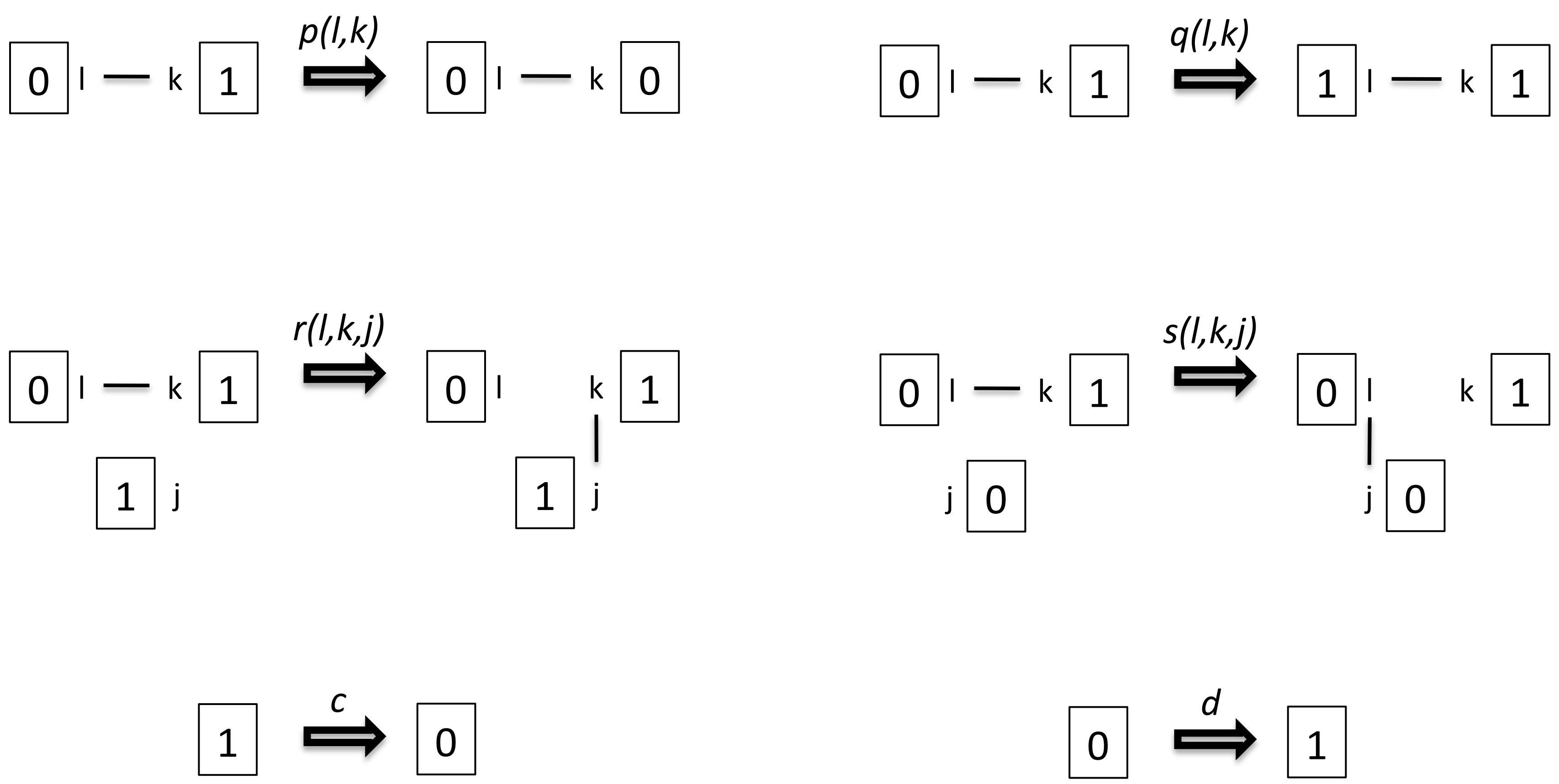


- Add labelled sites to act as ports to control degree, support rigidity
- Use diagrammatic notation

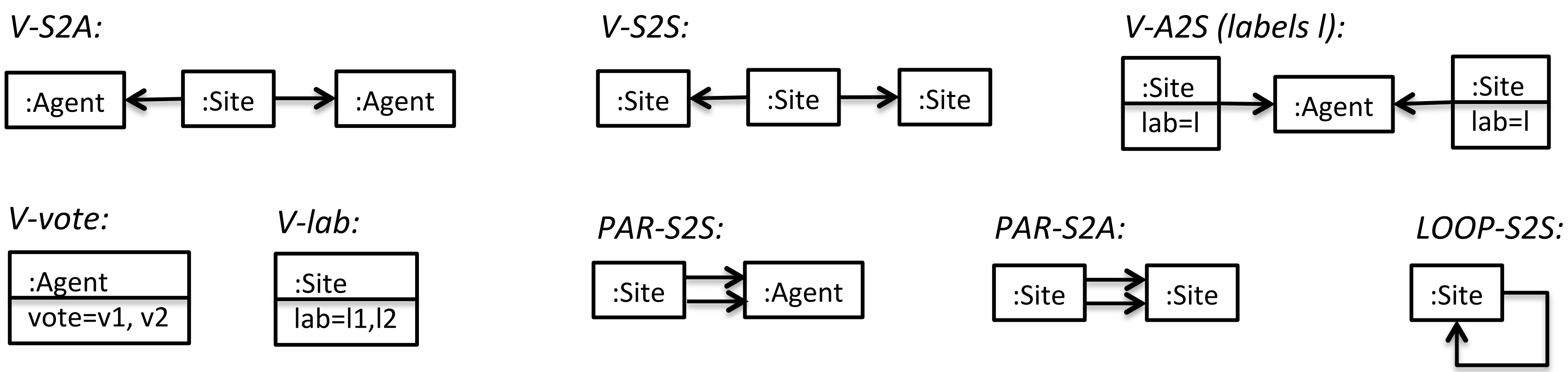




2. Explicit Model: Rules



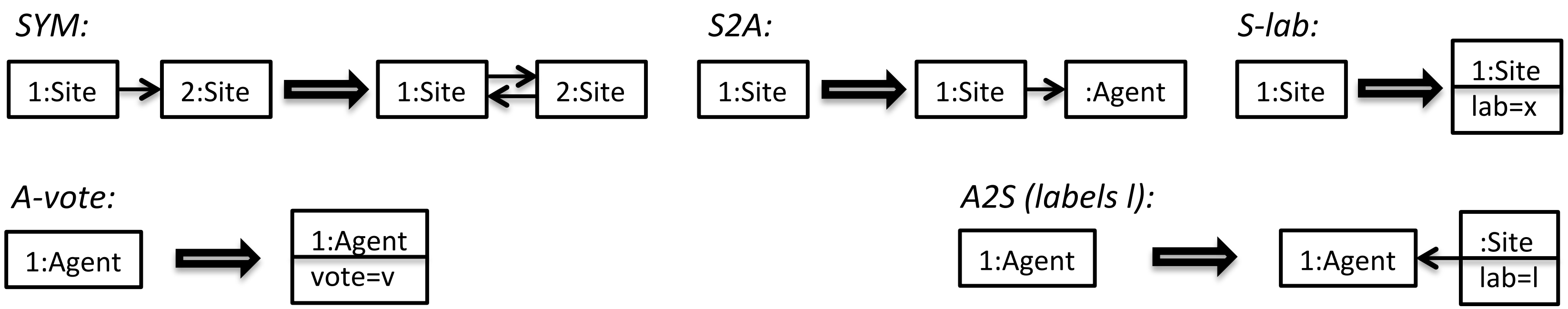
2. (Negative) Constraints on Patterns



Given adhesive ambient *category C of structures*, forbidden substructures S specify *full subcategory PC of patterns*

Taking as forbidden substructures all *non-rigid atomic structures* guarantees rigidity of *PC*

2. (Positive) Constraints on States



Patterns can be incomplete, states cannot

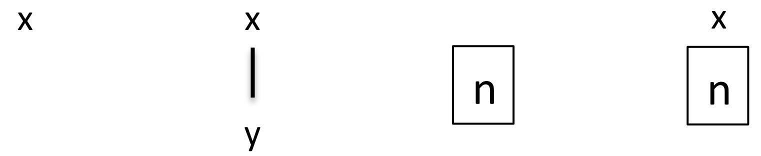
Category of states *SC full subcategory of PC*

3. Select Minimal Permissible Rules

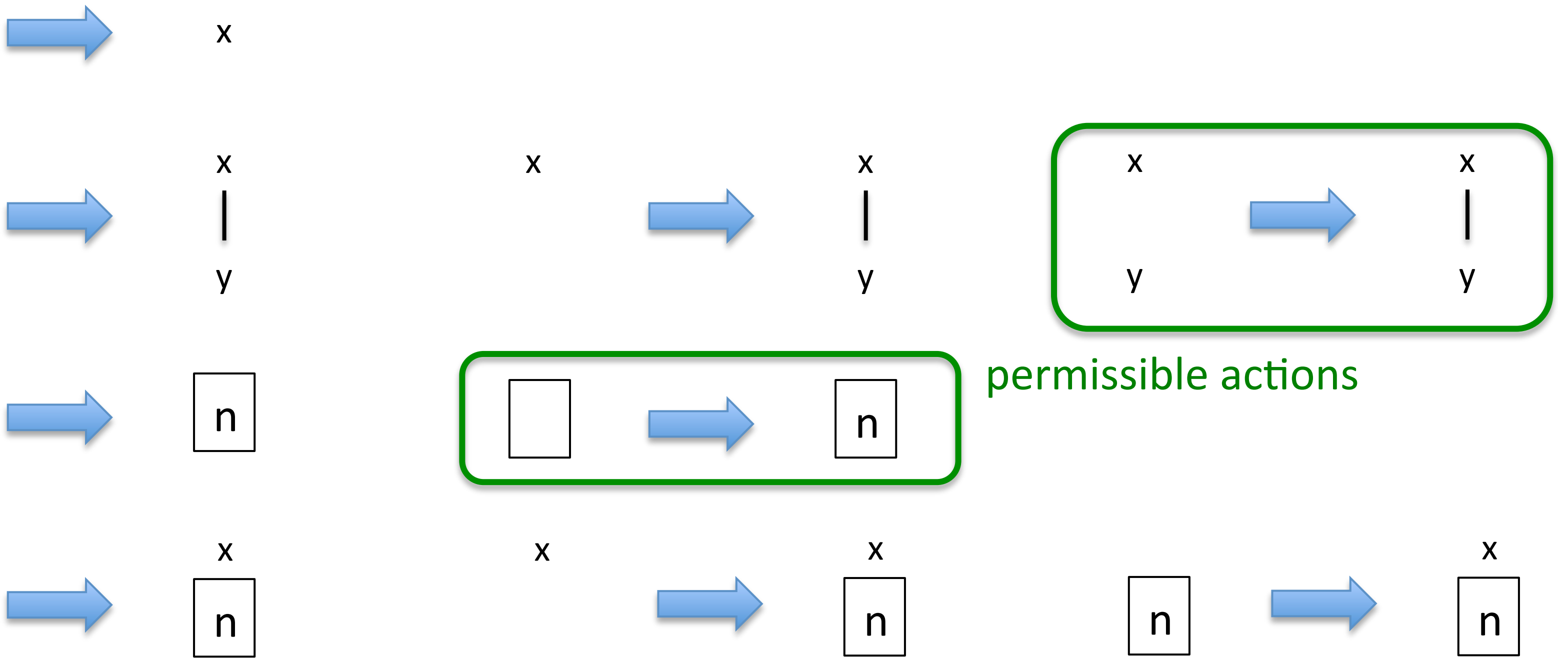
(creation only, delete actions symmetrical)

- Atomic graphs

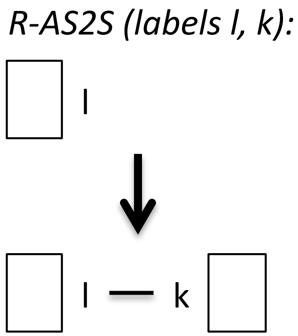
  x, y, z in {a, b, c} distinct

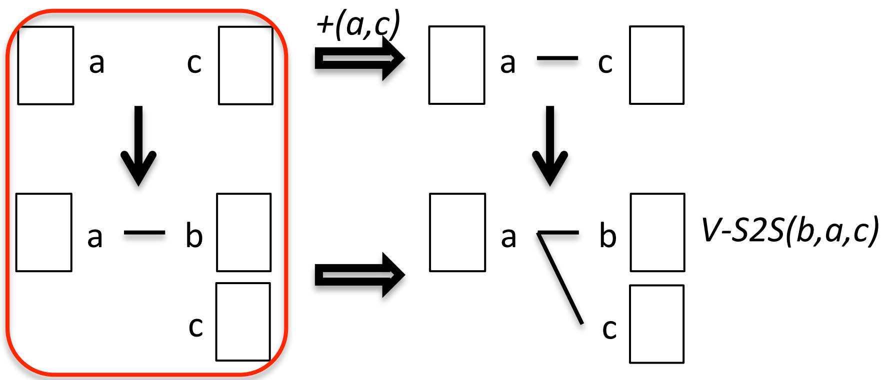


- Minimal creation rules:



4. Derive Constraints for Matches

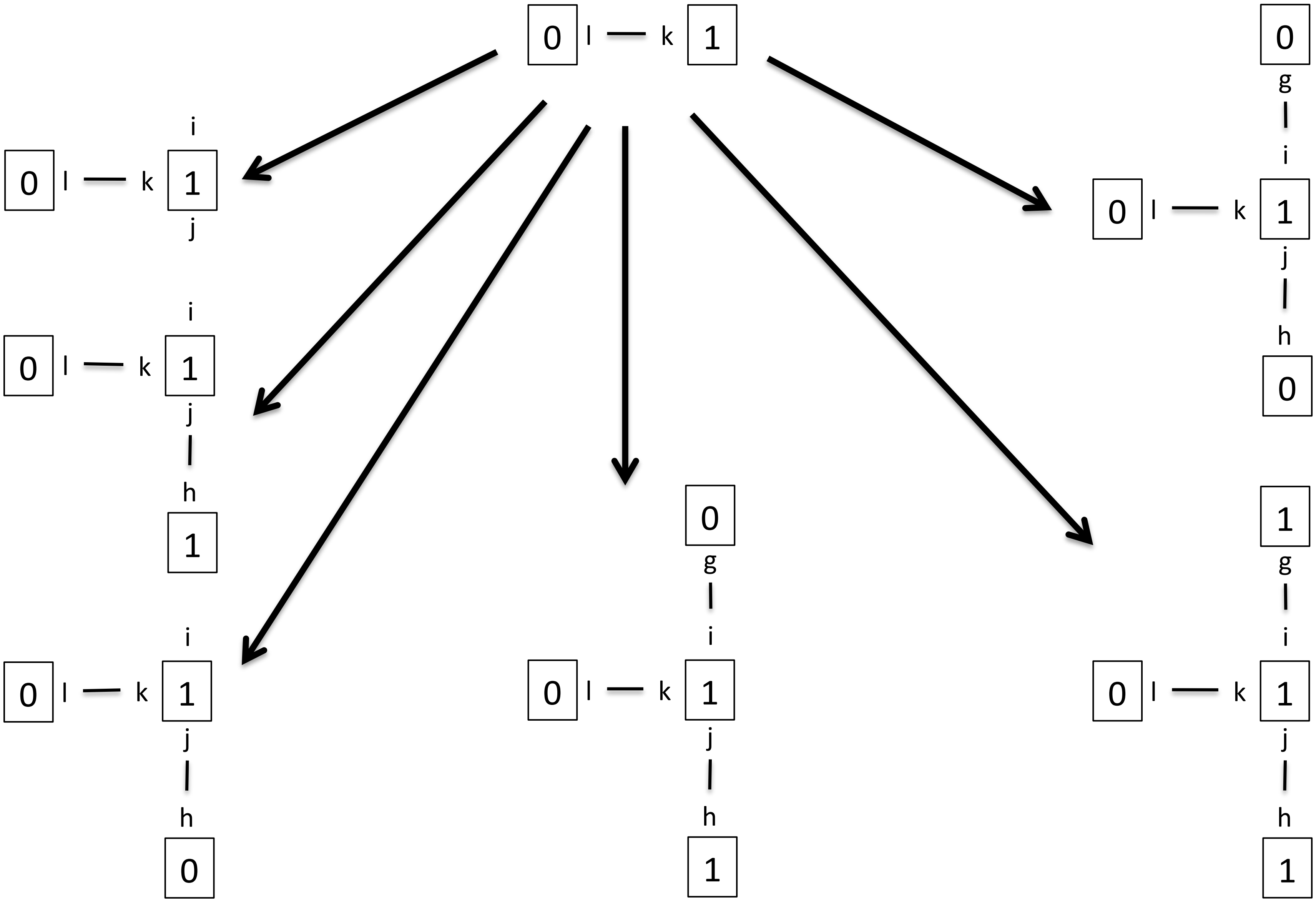


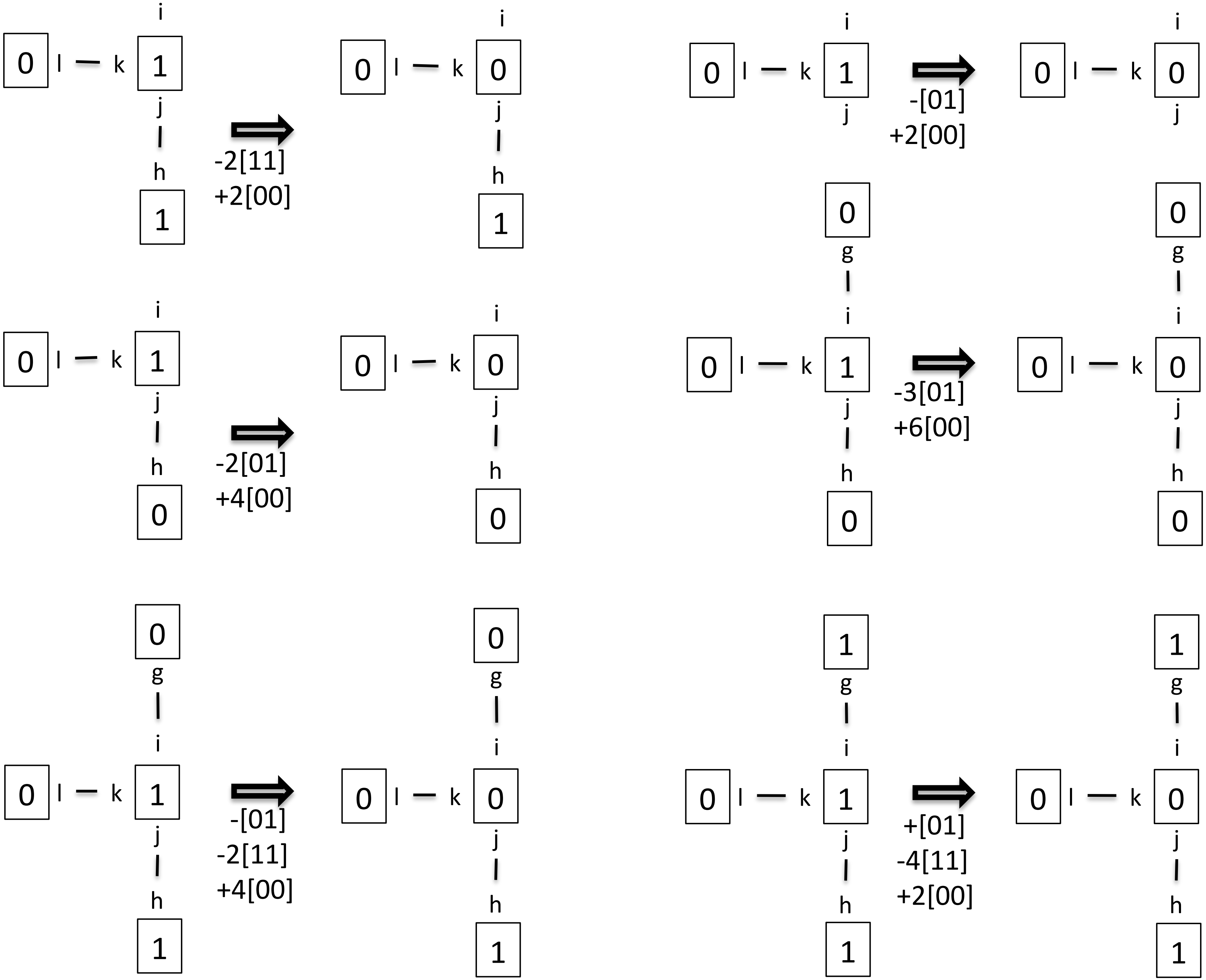


Path category, so that matches are *open maps*.

- ensure that negative pattern constrains are preserved
- control embedding into context

5. Petrify




-2[11]
+2[00]
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Methodology

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Conclusion

We can now analyse the resulting PT net by simulation, deriving differential equations, using it for parameter fitting, etc.

Under meaningful restrictions this works in general adhesive categories.

Questions:

- How to systematically rigidify an existing non-rigid model?
- How to handle attributes other than by instantiation?