An Overview of Recent Research Activities around CafeOBJ

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Topics of this talk

- Some introductory remarks on Formal Methods, CafeOBJ, and Proof Scores
- Combination of inference and search in the proof score method
  - Abstraction-with-Inference + search
  - Induction Guided Falsification
    - Backward-search (inference) + forward-search (search)
- Sound and complete proof rules underlie the proof score method
- Concluding remarks

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Our Perception on Formal Methods and Specification Verifications

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Application areas of formal methods (FM)

1. Analysis and verification of developed program codes (post-coding)

2. Analysis and verification of (models/specs of) domains, requirements, and designs before/ without coding (pre-coding or without coding)

Successful application of formal methods to the area of (modeling/specification of) domains, requirements, designs can bring drastic good effects for systems developments, but it is not well exploited and/or practiced yet.

specification = description of model

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The current situation of FM

- Verification with formal specifications still have a potential to improve the practices in upstream (pre-coding) of systems development processes
- Model checking has brought a big success but still has limitations
  - It is basically "model checking" for program codes
    - Still mainly for post-coding
  - Infinite state to finite state transformation can be unnatural and difficult
- Established interactive theorem provers (Isabelle/HOL, Coq, PVS, etc.) are still to be well accepted to ordinary software/systems engineers
  - especially in upstream (pre-coding) phase

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Our approach

- Reasonable blend of user and machine capabilities, intuition and rigor, high-level planning and tedious formal calculation
  - fully automated proofs/verifications are not necessary good for human beings to perceive logical structures of real problems/systems
  - interactive understanding/description of real problem domains/requirements/designs is necessary

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Proof Score Approach

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Proof Score
as a Complete Set of Symbolic Test cases

- Domain/requirement/design engineers are expected to construct proof scores together with formal specifications
- Proof score is a complete set of symbolic test cases such that when executed (or evaluated/ reduced) and everything evaluates as expected, then the desired property is convinced (or proved) to hold. Proof score is supposed to be read by engineers.
  - Proof by construction/development
  - Proof by computation/reduction/rewriting
  - Test Driven (Specification) Development

Development of proof scores in CafeOBJ

- Many simple proof scores are written in OBJ language from 1980's; some of them are not trivial
- From around 1997 CafeOBJ group at JAIST use proof scores seriously for verifying specifications for various examples
  - From static to dynamic/reactive system
  - From ad hoc to more systematic proof scores
  - Introduction of OTS (Observational Transition System) was a most an important step

Some achievements of CafeOBJ/OTS proof score approach

CafeOBJ/OTS approach has been applied to the following kinds of problems and found usable:

- Some classical mutual exclusion algorithms
- Some real time algorithms
  e.g. Fischer's mutual exclusion protocol
- Railway signaling systems
- Authentication protocol
  e.g. NSLPK, Otway-Rees, STS protocols
- Practical sized e-commerce protocol of SET
  (some of proof score exceeds 60,000 lines;
  specification is about 2,000 lines,
  20-30 minutes for reduction of the proof score)
- UML semantics (class diagram + OCL-assertions)
- Formal Fault Tree Analysis
- Secure workflow models, internal control

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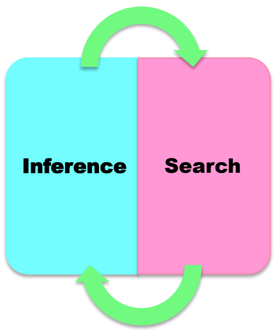
Verification by Inference and Search in Proof Scores



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Two topics

- Abstraction by inference (TP) and counter example finding by search (MC)
  - QLOCK example
- Counter example finding by MC (Search) and TP (Inference)
  - NSPK example

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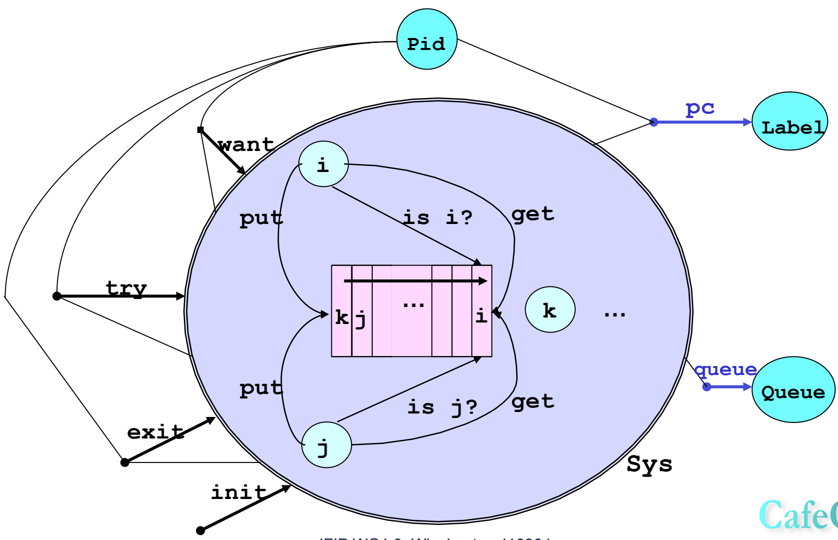
Modeling QLOCK (via Signature Diagram) with OTS (Observational Transition System)



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CafeOBJ signature for QLOCKwithOTS

```
-- state space of the system
*[Sys]*                                        system sort declaration

-- visible sorts for observation
[Queue Pid Label]                              visible sort declaration

-- observations
bop pc : Sys Pid -> Label                      observation declaration
bop queue : Sys -> Queue

-- any initial state
bop init : -> Sys {constr}                     initial state declaration
-- actions
bop want : Sys Pid -> Sys {constr}
bop try  : Sys Pid -> Sys {constr}             action declaration
bop exit : Sys Pid -> Sys {constr}
```

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Transition system for QLOCK (1)

```
mod* QLOCKconfig {
  inc(QLOCK)
  [ Config ]
  op <_> : Sys -> Config .
}
-- pre-transiton system with an agent/process p
mod* QLOCKpTrans {
  inc(QLOCKconfig)
  op p : -> PidConst .
  var S : Sys .
  -- possible transitions
  ctrans < S > => < want(S,p) > if c-want(S,p) .
  ctrans < S > => < try(S,p) >  if c-try(S,p) .
  ctrans < S > => < exit(S,p) > if c-exit(S,p) .
}
```

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Transition system for QLOCK (2)

```
-- transition system with 2 agents i j
mod* QLOCKijTrans {
  inc((QLOCKpTrans * {op p -> i}) +
      (QLOCKpTrans * {op p -> j}))
}

-- transition system with of 3 agents i j k
mod* QLOCKijkTrans {
  inc(QLOCKijTrans +
      (QLOCKpTrans * {op p -> k}))
}
```

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Search predicate of CafeOBJ a la Maude's search command

CafeOBJ System has the following built-in predicate:
- **Any** is any sort (that is, the command is available for any sort)
- **NzNat*** is a built-in sort containing non-zero natural number and the special symbol "*" which stands for infinity

```
pred _=(_,_)=>*_ : Any NzNat* NzNat* Any
```

`(t1 =(m,n)=>* t2)` returns **true** if `t1` can be translated (or rewritten), via more than 0 times transitions, to some term which matches to `t2`. Otherwise, it returns **false**. Possible transitions/rewritings are searched in breadth first fashion. `n` is upper bound of the depth of the search, and `m` is upper bound of the number of terms which match to `t2`. If either of the depth of the search or the number of the matched terms reaches to the upper bound, the search stops.

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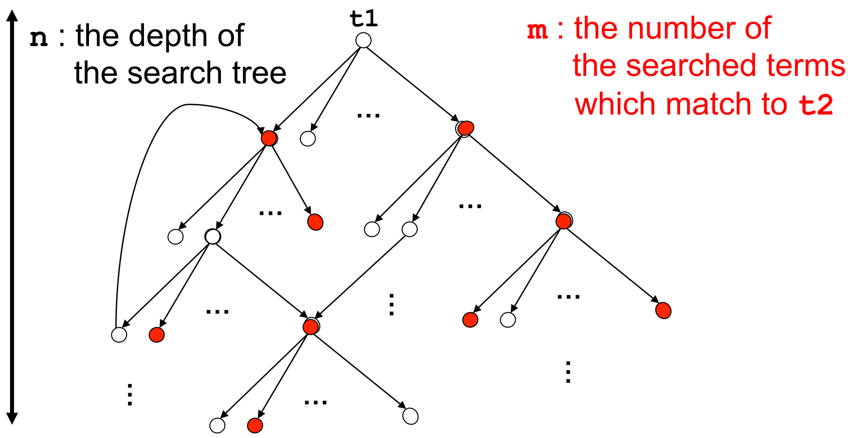
t1 =(m,n)=>* t2



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suchThat predicate

`t1 =(m,n)=>* t2 suchThat pred1(t2)`

pred1(t2) is a predicate about **t2** and can refer to the variables which appear in **t2**.
pred1(t2) enhances the condition used to determine the term which matches to **t2**.

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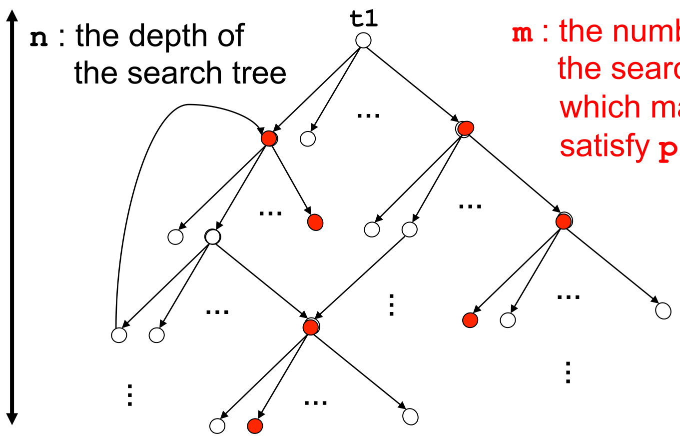
t1 =(m,n)=>* t2 suchThat pred1(t2)

n : the depth of the search tree



m : the number of the searched terms which match to **t2** and satisfy **pred(t2)**

`withStateEq` predicate

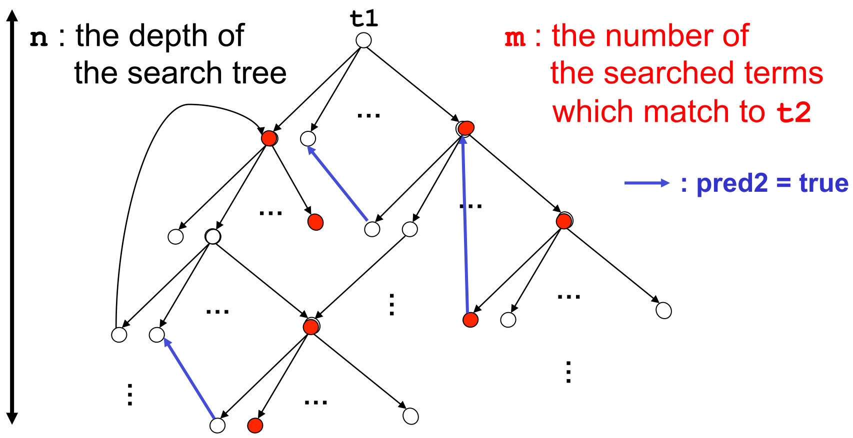
```
t1 =(m,n)=>* t2
   withStateEq pred2(V1:St,V2:St)
```

Pred2(V1:St,V2:St) is a binary predicate of two arguments with the same sort `St` of the term **t2**.
Pred2(V1:St,V2:St) is used to determine a newly searched term (a state configuration) is already searched one.
If this **withStateEq** predicate is not given, the term identity binary predicate is used for the purpose.

Using both of `suchTant` and `withStateEq` is also possible

```
t1 =(m,n)=>* t2 suchThat pred1(t2)
             withStateEq pred2(S1:Sort,S2:Sort)
```

t1 =(m,n)=>* t2
withStateEq pred2(V1:St,V2:St)



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Verification by Searching with Observational Equivalence

```
red in (QLOCKijTrans + QLOCKobEq + MEX) :
       < init > =(*,*)=>* < S:Sys >
       suchThat (not mutualEx(S,i,j))
       withStateEq (C1:Config =ob= C2:Config) .
```

This CafeOBJ code searches for a counter example of mutual exclusion property in the whole state space Sys(i,j) of two agents system. If this returns `false`, the two agents system is verified to have the mutual exclusion property.

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Simulation of any number of agents systems by the two agents system

Proof scores of simOfQLOCKbyQLOCKijPS.mod csQtopPS.mod

verify the following

let i and j be any two distinctive process identifiers, and let Sys(i,j) be the state space of QLOCK with only the two processes i and j,
then
(there is a counter example in Sys)
implies
(there exits a counter example in Sys(i,j))
that is,
(for-all t:Sys(i,j)).pred(t,i,j)
implies
(for-all s:Sys).pred(s,i,j)

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Counter example finding by forward and backward search -- another kind of collaborative use of MC & TP

- MC & TP can be collaboratively used to find a counterexample that exists at a deep position.
  - Properties concerned are *invariants*.
  - *Bounded model checking* (*BMC*) is used as an MC technique.
  - *Induction* is mainly used as a TP technique.
- We have proposed a collaborative use of BMC & induction to find a deep counterexample for invariants: *Induction-Guided Falsification* (*IGF*).

K. Ogata, M. Nakano, W. Kong, K. Futatsugi: Induction-Guided Falsification, 8th ICFEM, LNCS 4260, Springer, pp.114-131 (2006).

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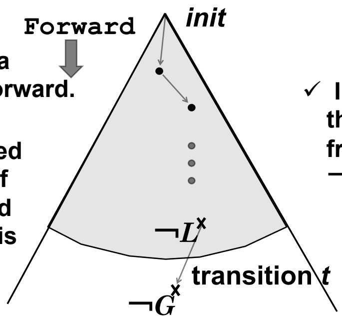
Induction-Guided Falsification (IGF)

- ✓ Suppose that a counterexample of an invariant *G* exists outside of the bounded reachable state space that can be exhaustively traversed.
- ✓ induction may conjecture a lemma *L* such that its counterexample exists in the space.

- ✓ BMC tries to find a counterexample forward.
- ✓ IGF can be regarded as a combination of forward & backward reachability analysis methods.



- ✓ Induction tries to show that there are no paths from any states such that ¬*G* to any initial states.

Backward

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Sound and “Complete” Proof Rules for Proof Scores

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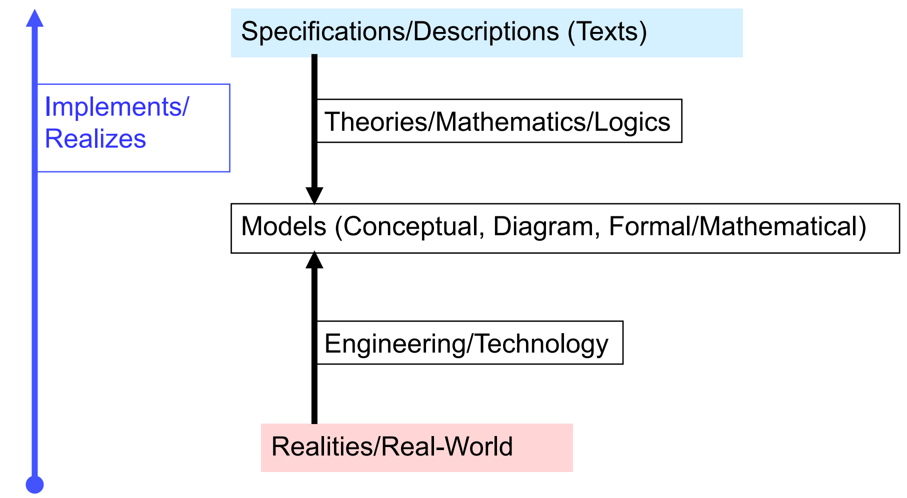
Topics

- Specification/Descriptions, Models, and Realities
- Constructor-based Order Sorted Algebra
- Satisfaction of a Property by a Specification
  - SPEC |= prop
- Proof rules for SPEC |= prop and SPEC |- prop

Specifications, Models, Realities



Specification

An **constructor-based equational specification SPEC** in CafeOBJ (a text in the CafeOBJ language with only equational axioms) is defined as a pair **(Sig,E)** of order-sorted constructor-based signature **Sig** and a set **E** of conditional equations over **Sig**. A signature **Sig** is defined as a triple **(S,F,F^c)** of an partially ordered set **S** of sorts, an indexed family **F** of sets of **S**-sorted functions/operations, and a set **F^c** of constructors. **F^c** is a family of subsets of **F**, i.e. $\mathbf{F^c} \subseteq \mathbf{F}$.

SPEC = ((S,F,F^c),E)

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Model: (S,F)-Algebra

A formal/mathematical **model** of a specification **SPEC = ((S,F,F^c),E)** is an reachable order-sorted **algebra A** which has the signature **(S,F)** and satisfies all equations in **E**.

An order-sorted algebra which has a signature (S,F) is called an **(S,F)-algebra.** An **(S,F)-algebra A** interprets a sort symbol s in **S** as a (non empty) set $\mathbf{A_s}$ and an operation (function) symbol **f :s1 s2 …sn->s(n+1)** in **F** as a function $\mathbf{A_f : A_{s1},A_{s2},..,A_{sn} \rightarrow A_{s(n+1)}}$. The interpretation respects the order-sort constrains.

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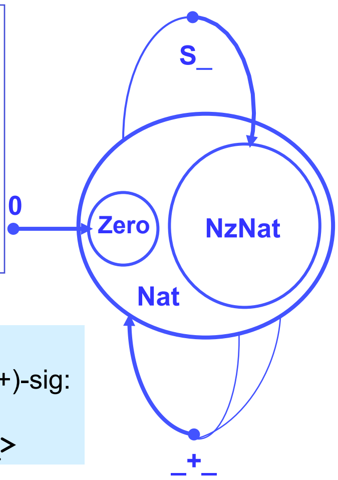
An example of Signature and its Algebra

```
-- Let (PNAT+)-sig be
-- the  signature of PNAT+
-- sort
[ Zero NzNat < Nat ]
-- operators
op 0 :   -> Nat {constr}
op s_ :  Nat -> NzNat {constr}
op _+_ : Nat Nat -> Nat
```



A (PNAT+)-sig-algebra
Order-Sorted Algebra with Signature (PNAT+)-sig:

`<Nat, NzNat, Zero; 0, s_, _+_>`

Model: (S,F,F^C)-Algebra

If a sort s ∈ S is the co-arity of some operator f ∈ F^C, the sort s is called a **constrained sort**. A sort which is not constrained is called a **loose sort**.

An (S,F)-algebra A is called **(S,F,F^C)-algebra** if any value v ∈ A_s for any constrained sort s ∈ S is expressible only using
(1) function A_f for f ∈ F^C
and
(2) function A_g for g ∈ F whose co-arity is loose sort .

(S,F,F^C)-algebra can also be called **F^C-reachable algebra**

Valuation, Evaluation

A **valuation** (or an assignment) is a sort preserving map from the (order-sorted) set of variables of a specification to an order-sorted algebra (a model), and assigns values to all variables.

Given a model **A** and a valuation **v**, a **term t** of sort **s,** which may contain variables, is evaluated to a **value** $\mathbf{A_v(t)}$ in $\mathbf{A_s}$

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Equation

Given terms t, t', t_1, t_1', t_2, t_2'…t_n, t_n' , a **conditional equation** is a sentence of the form:

$$\mathbf{t = t' \text{ if } (t_1 = t_1') \wedge (t_2 = t_2') \wedge \ldots \wedge (t_n = t_n')}$$

An ordinary equation is a sentence of the form:

t = t'

that is n=0.

A conditional equation in CafeOBJ notation:

t = t' if c

where t,t' are any terms and c is a Boolean term is an abbreviation of

t = t' if c = true

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Satisfiability of Equation

An ordered-sorted algebra **A** **satisfies** a conditional equation:

$t = t'$ if $(t_1 = t_1')$ /\ $(t_2 = t_2')$ /\…/\ $(t_n = t_n')$

iff

$A_v(t_1)=A_v(t_1')$ and $A_v(t_2)=A_v(t_2')$ and…and $A_v(t_n)=A_v(t_n')$ implies $A_v(t)=A_v(t')$

for any valuation **v** .

The satisfaction of an equation by a model **A** is denoted by
A |= ($t = t'$ if $(t_1 = t_1')$ /\ $(t_2 = t_2')$ /\…/\ $(t_n = t_n')$)

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CafeOBJ _=_ (meta-level equality) and Boolean _=_ (object-level equality)

If a specification SP includes,

```
op _=_ : S S -> Bool .
eq (X = X) = true .
ceq X = Y if (X = Y) .
```

then

SP |= t=t' if $(t_1=t_1')$/\ $(t_2=t_2')$/\…/\$(t_n=t_n')$

iff

SP |= ((t_1=t_1' and t_2=t_2' and …and t_n=t_n') implies t=t') = true .

1. **Object-level equality can substitute for meta-level equality**
2. **Every sentence (conditional equation) can be written as a Boolean term.**

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SPEC-algebra

For a specification **SPEC** = ((**S,F,F^c**), **E**), a **SPEC-algebra** is a **(S,F,F^C)-algebra** which satisfies all equations in **E** .

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Satisfiability of property by specification: SPEC |= prop

A specification **SPEC** = **((S,F,F^c),E)** is defined to satisfy a property **p** (a term of sort **Bool)** iff **A |= (p** = true) holes for any **SPEC-algebra A.**

The satisfaction of a predicate **prop** by a specification **SPEC** = **((S,F,F^c),E)** is denoted by:

SPEC |= p or **E |= p**

A most important purpose of developing a specification **SPEC = ((S,F,F^c),E)** in CafeOBJ is to check whether

SPEC |= prop

holds for a predicate **prop** which describes some important property of the system which **SPEC** specifies.

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Proof rules for SPEC |= prop (semantic entailment)

For doing formal verification, it is common to think of syntactic (proof theoretic) entailment:

SPEC |- prop

which corresponds to semantic entailment:

SPEC |= prop .

We have developed a sound and *quasi* complete set of proof rules for |- which satisfies:

SPEC |- prop iff SPEC |= prop

for unstructured specifications and constitutes a theoretical foundation for verifications with proof scores.

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Proof Rules (1) -- entailment system (S, P, or Ei denotes a set of equations)

Monotonicity: $\dfrac{}{E1 \mid\!- E2}$ for any $E2 \subseteq E1$

Transitivity: $\dfrac{E1 \mid\!- E2 ,\ E2 \mid\!- E3}{E1 \mid\!- E3}$

Unions: $\dfrac{E1 \mid\!- E2 ,\ E1 \mid\!- E3}{E1 \mid\!- E2\ U\ E3}$

Translation: $\dfrac{S \mid\!-_{\Sigma} P}{\varphi(S) \mid\!-_{\Sigma'} \varphi(P)}$ for any signature morphism $\varphi: \Sigma \to \Sigma'$

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Proof Rules (2) -- equational reasoning

(t and ti denotes terms, f denotes operator, p denotes predicate)

Reflexivity: $\dfrac{}{\text{|- \{t=t\}}}$

Symmetry: $\dfrac{}{\text{\{t1=t2\} |- \{t2=t1\}}}$

Transitivity: $\dfrac{}{\text{\{t1=t2, t2=t3\} |- \{t1=t3\}}}$

Congruence: $\dfrac{}{\text{\{t1=t1',t2=t2',...,tn=tn'\} |- f(t1,t2,...,tn)=f(t1',t2',...,tn')}}$

P-Congruence: $\dfrac{}{\text{\{t1=t1',t2=t2',...,tn=tn'\} U \{p(t1,t2,...,tn)\} |- p(t1',t2',...,tn')}}$

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Proof Rules (3)

(H denotes a set of equations, p denotes predicate, X, Y, or Z denotes set of variables, x denotes variable)

Implication: $\dfrac{\text{S |- P U \{(/\textbackslash H=>p)\}}}{\text{S U H |- P U \{p\}}}$ and $\dfrac{\text{S U H |- P U \{p\}}}{\text{S |- P U \{(/\textbackslash H=>p)\}}}$

Substitutivity: $\dfrac{\text{S |- P}}{\text{S U \{(}\forall\text{x)p\} |- P U \{(}\forall\text{Y)p(x<-t)\}}}$

Generalization: $\dfrac{\text{S } |\text{-}_{\Sigma} \text{ P U \{(}\forall\text{Z)p\}}}{\text{S } |\text{-}_{\Sigma(Z)} \text{ P U \{p\}}}$ and $\dfrac{\text{S } |\text{-}_{\Sigma(Z)} \text{ P U \{p\}}}{\text{S } |\text{-}_{\Sigma} \text{ P U \{(}\forall\text{Z)p\}}}$

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Proof Rules (4) – these are infinite in nature

(p denotes predicate, Y denotes set of variables, x denotes variable, f denotes a function, ti denotes a term)

C-Abstraction (Constructor Abstraction):

$$\frac{\{\ (S \vdash \{(\forall Y)p(x{<}{-}t)\})\ \mid\ t \text{ is constructor Y-term, Y are loose vars}\ \}}{S \vdash \{(\forall x)p\}}$$

Case Analysis:

$$\frac{\{\ (S \cup \{f(t1,\ldots,tn)=t\} \vdash_{\Sigma(Y)} \{p\})\ \mid\ t \text{ is constructor Y-term, Y are loose vars}\ \}}{S \vdash_{\Sigma} \{p\}}$$

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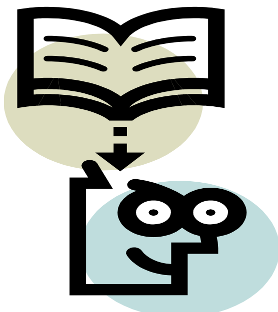
Concluding Remarks



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Three levels of CafeOBJ applications

1. Construct formal models; describe formal specifications
2. Do rapid prototypings or animations and check the properties of specifications; execute specifications for validations/verifications
3. Write proof scores to verify properties of specifications; verifications/proofs with reductions/ rewritings

Choose an appropriate level
depending on problems and situations

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Prerequisites for proof score writing in CafeOBJ (1)

- Algebraic modeling: development of algebraic specifications
  - defining signature for a real problem
  - expressing the semantics of a problem in equations
    - more exactly, expressing the problem in reduction rules

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Prerequisites for proof score writing in CafeOBJ (2)

- Equational logic, rewriting, and propositional calculus
  - equationl reasoning
    - equivalence relation, equational calculus, …
  - propositional calculus with “xor” normal forms which has the complete rewriting calculus
  - reduction/rewriting
    - termination, confluence, sufficiently completeness

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Prerequisites for proof score writing in CafeOBJ (3)

- Proof by induction and case analysis
  - case splitting using constructors or key predicates in specifications
  - discovery of lemmas
  - decomposition of a goal predicate into an appropriate conjunctive form

These are the most difficult parts of proof score writing

But this is common to any kind of interactive verifiers!

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Traceability in proof score approach with CafeOBJ

- All reductions are done exactly using equations in specifications as rewriting rules
  - this make it easy to detect necessary changes in specs for letting something happen (or not happen)
- Usually reductions are sufficiently fast, and encourage prompt interactions between user and system

This is a quit unique feature of the proof score approach with CafeOBJ comparing to other verification method which often involves several formalisms/logics and translations between them

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Equational proofs by reduction/rewriting

Why do we care about "equational reasoning by reduction" ?

- It is simple and powerful and a promising light weighted formal reasoning method
  - easy to understand and can be more acceptable for software engineers
- It supports transparent relation between specs and reasoning by reduction (good traceability)

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Future Issues

- Development of the environment for proof score constructions
  - Standard platforms for programming environment can be naturally used
  - Proof score checker to check correctness of the proof scores as independently as possible
  - Farther development of the Kumo/Tatami scheme to realize a web (or hypertext) based constructions of specs and proof scores
- Serious development of practical domain/requirement/design specifications in the application area like e-government, e-commerce, open standards for automotive software, etc.
  - The development should aim at reasonable balance of informal and the formal specifications, and verify as much as meaningful and important properties of the models/problems the specifications are describing

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CafeOBJ official home page

http://www.ldl.jaist.ac.jp/cafeobj/

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