Symbolic Executions of Epistemic Ensembles

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A Simple Bit Transmission Protocol

Epistemic ensemble of epistemic processes executing epistemic actions guarded by epistemic formulæ

$$
\begin{aligned}
\mathit{Ag1} &= \mu X \,.\, \big(\neg \mathsf{K}_1\, \mathsf{K}_2\, x_1 \supset \mathit{snd}^{1\to 2}_{\mathsf{los}}(x_1).X + \\
&\qquad\qquad \mathsf{K}_1\, \mathsf{K}_2\, x_1 \supset \mathbf{1.0}\big) \\
\mathit{Ag2} &= \mathsf{K}_2\, x_1 \supset \mathit{snd}^{2\to 1}_{\mathsf{rel}}(\mathsf{K}_2\, x_1).\mathbf{0} \\
\mathit{Ag3} &= \mathsf{K}_3\, x_1 \supset \mathit{snd}^{3\to 1}_{\mathsf{rel}}(\mathsf{K}_3\, x_1).\mathbf{0} \\
\mathit{Sys} &= 1 : \mathit{Ag1} \parallel 2 : \mathit{Ag2} \parallel 3 : \mathit{Ag3}
\end{aligned}
$$

- A. Baltag, L. S. Moss. Logics for epistemic programs. Synthese, 2004.
- E. Lorini, E. Perrotin, F. Schwarzentruber. Epistemic actions: Comparing multi-agent belief bases with action models. KR, 2022.
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Goal: Executing ensembles concretely, symbolically, and distributed

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Executing Epistemic Ensembles

- Concretely on a set of epistemic states

$$a : P_a \parallel \vec{E}, \mathcal{K} \xrightarrow{\alpha} a : P'_a \parallel \vec{E}, \mathcal{K}'$$

  - infinite state space

- Symbolically on a global knowledge base

$$a : P_a \parallel \vec{E}, \Gamma \xrightarrow{\alpha}{}^{\Delta} a : P'_a \parallel \vec{E}, \Gamma'$$

  - finite state space for model checking due to abstractions

- Distributed on agent-wise local knowledge bases

$$i : (P_i, \Lambda_i) \parallel (a : (P_a, \Lambda_a))_{a \in C} \parallel \vec{D} \xrightarrow{\alpha}{}^{\vec{\nabla}} \\ i : (P'_i, \Lambda'_i) \parallel (a : (P_a, \Lambda'_a))_{a \in C} \parallel \vec{D}$$

  - parallel execution, finite state space

Epistemic Logic

$$\mathscr{F} \ni \varphi ::= p \mid \mathrm{true} \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathsf{K}_a\,\varphi$$

- set of propositions $\Pi \ni p$, (finite) set of agents $A \ni a$
- $\mathsf{K}_a\,\varphi$ read as "agent a knows φ"

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Interpreted over epistemic structure $K = (W, (E_a)_{a \in A}, L)$

- worlds W, labelling $L\colon W \to \wp\Pi$
- accessibility relation $E_a \subseteq W \times W$ of possible worlds for each agent a
  - equivalence relation for **S5**

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  - equivalence relation for **S5**

Satisfaction relation $K, w \models \varphi$ with $w \in W$

$$K, w \models \mathsf{K}_a\,\varphi \iff K, w' \models \varphi \text{ f. a. } w' \in W \text{ with } (w, w') \in E_a$$

Epistemic Logic: Axioms

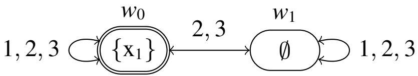
$\mathbf{S5}$-logic — all E_a equivalence relations

- $(\mathsf{K}_a\,\varphi_1 \wedge \mathsf{K}_a(\varphi_1 \to \varphi_2)) \to \mathsf{K}_a\,\varphi_2$ — distribution axiom
- $\mathsf{K}_a\,\varphi \to \varphi$ — knowledge axiom
  - "Known facts are true"
  - E_a reflexive on W
- $\mathsf{K}_a\,\varphi \to \mathsf{K}_a\,\mathsf{K}_a\,\varphi$ — positive introspection axiom
  - "If agent a knows φ, then it knows that it knows φ"
  - E_a transitive
- $\neg\mathsf{K}_a\,\varphi \to \mathsf{K}_a\,\neg\mathsf{K}_a\,\varphi$ — negative introspection axiom
  - "If agent a does not know φ, then it knows that it does not know φ"
  - E_a Euclidean
- $\neg\mathsf{K}_a\,\mathrm{false}$ — consistency axiom
  - "No agent believes false"
  - E_a serial

Epistemic States

Epistemic state $\mathfrak{K} = (K, w)$

Example Epistemic state $\mathfrak{K}_0$



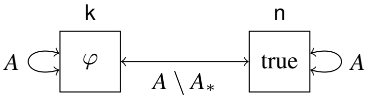
- $\mathfrak{K}_0 \models \mathsf{K}_1\, x_1, \quad \mathfrak{K}_0 \models \neg\mathsf{K}_2\, x_1, \quad \mathfrak{K}_0 \models \neg\mathsf{K}_3\, x_1$
- $\mathfrak{K}_0 \models \mathsf{K}_1\, \neg\mathsf{K}_2\, x_1, \quad \mathfrak{K}_0 \models \mathsf{K}_1\, \neg\mathsf{K}_3\, x_1$

Epistemic Actions

Epistemic action model $U = (Q, (F_a)_{a \in A}, pre)$

- events Q
- accessibility $F_a \subseteq Q \times Q$
- precondition $pre\colon Q \to \mathscr{F}$

Example Group announcement $U_{grp}(A_*, \varphi)$



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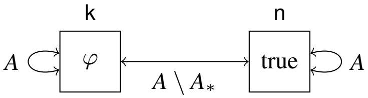

Epistemic action $\mathfrak{u} = (U, q)$ $pre(\mathfrak{u}) = pre(q)$

Epistemic Updates

Product update $(K, w) \lhd (U, q) = (K \lhd U, (w, q))$ if $K, w \models pre(q)$

$$(W', E', L') = (W, E, L) \lhd (Q, F, pre)$$
$$W' = \{(w, q) \in W \times Q \mid K, w \models pre(q)\}$$
$$E'_a = \{((w, q), (w', q')) \mid (w, w') \in E_a,\ (q, q') \in F_a\}$$
$$L'(w, q) = L(w)$$

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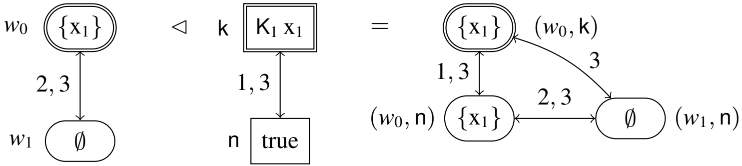
Example $\mathfrak{K}_1 = \mathfrak{K}_0 \lhd (U_{grp}(\{2\}, \mathsf{K}_1\, x_1), \mathsf{k})$



- $\mathfrak{K}_1 \models \mathsf{K}_2\, \mathsf{K}_1\, x_1$, but $\mathfrak{K}_1 \models \neg \mathsf{K}_1\, \mathsf{K}_2\, \mathsf{K}_1\, x_1$

Composite Epistemic Actions

Epistemic choice actions $\alpha = \{\mathfrak{u}_1, \ldots, \mathfrak{u}_n\}$ $\qquad pre(\alpha) = \bigvee_{\mathfrak{u} \in \alpha} pre(\mathfrak{u})$

- choice dependent on current state, according to precondition

Examples

$$\alpha^{\mathsf{k}}_{x_1} = \{(U_{grp}(\{2\}, \mathsf{K}_1\, \mathrm{x}_1), \mathsf{k}), (U_{grp}(\{2\}, \mathsf{K}_1\, \neg \mathrm{x}_1), \mathsf{k})\}$$
$$\alpha^{\mathsf{n}}_{\mathrm{x}_1} = \{(U_{grp}(\{2\}, \mathsf{K}_1\, \mathrm{x}_1), \mathsf{n})\}$$
$$\alpha^{\mathsf{n}}_{\neg \mathrm{x}_1} = \{(U_{grp}(\{2\}, \mathsf{K}_1\, \neg \mathrm{x}_1), \mathsf{n})\}$$

$$snd^{1\to 2}_{\mathsf{los}}(\mathsf{K}_1\, x_1).X = \alpha^{\mathsf{k}}_{x_1}.X + \alpha^{\mathsf{n}}_{\mathrm{x}_1}.X + \alpha^{\mathsf{n}}_{\neg \mathrm{x}_1}.X$$
$$snd^{2\to 1}_{\mathsf{rel}}(\mathsf{K}_2\, x_1).X = \{(U_{grp}(\{1, 2\}, \mathsf{K}_2\, x_1), \mathsf{k})\}.X$$

($\mathsf{K}_a\, x$ abbreviates $\mathsf{K}_a\, \mathrm{x} \vee \mathsf{K}_a\, \neg \mathrm{x}$)

Concrete Epistemic States

$\mathcal{K}$ set of epistemic states

- $\mathcal{K} \models \varphi \iff \mathfrak{K} \models \varphi$ f. a. $\mathfrak{K} \in \mathcal{K}$

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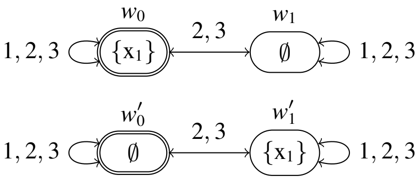
Example $\mathcal{K}_0 = \{\mathfrak{K}_0, \mathfrak{K}'_0\}$



- $\mathcal{K}_0 \models \mathsf{K}_1\, x_1 \vee \mathsf{K}_1\, \neg x_1$
- $\mathcal{K}_0 \not\models \mathsf{K}_2\, \mathsf{M}_1\, x_1 \vee \mathsf{K}_2\, \mathsf{M}_1\, \neg x_1$
  - $\mathsf{M}_a\, \varphi = \neg \mathsf{K}_a\, \neg \varphi$

Epistemic Processes on Concrete States

$$P ::= \mathbf{0} \mid \alpha.P \mid \varphi \supset P \mid P_1 + P_2 \mid \mu X\,.\,P \mid X$$

Concrete operational semantics on a set $\mathcal{K}$ of epistemic states

$\mathcal{K} \models \varphi \iff \mathfrak{K} \models \varphi$ f. a. $\mathfrak{K} \in \mathcal{K}$
$\mathcal{K} \lhd \alpha = \bigcup_{\mathfrak{u} \in \alpha} \mathcal{K} \lhd \mathfrak{u}, \qquad \mathcal{K} \lhd \mathfrak{u} = \{\mathfrak{K} \lhd \mathfrak{u} \mid \mathfrak{K} \in \mathcal{K}, \mathfrak{K} \models \mathit{pre}(\mathfrak{u})\}$

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$$\alpha.P, \mathcal{K} \xrightarrow{\alpha} P, \mathcal{K} \lhd \alpha \quad \text{if } \mathcal{K} \models \mathit{pre}(\alpha)$$

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$$\alpha.P, \mathcal{K} \xrightarrow{\alpha} P, \mathcal{K} \lhd \alpha \quad \text{if } \mathcal{K} \models \mathit{pre}(\alpha)$$

$$\frac{P, \mathcal{K} \xrightarrow{\alpha} P', \mathcal{K}'}{\varphi \supset P \xrightarrow{\alpha} P', \mathcal{K}'} \quad \text{if } \mathcal{K} \models \varphi$$

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$$\frac{P, \mathcal{K} \xrightarrow{\alpha} P', \mathcal{K}'}{\varphi \supset P \xrightarrow{\alpha} P', \mathcal{K}'} \quad \text{if } \mathcal{K} \models \varphi$$

$$\frac{P\{X \mapsto \mu X \,.\, P\}, \mathcal{K} \xrightarrow{\alpha} P', \mathcal{K}'}{\mu X \,.\, P \xrightarrow{\alpha} P', \mathcal{K}'}$$

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Epistemic Ensembles on Concrete States

For $G \subseteq A$, G-families of epistemic processes $\vec{E} = (a : P_a)_{a \in G}$

- each process P_a only contains (pre-)conditions in $\mathscr{F}{\restriction}a$

$\mathscr{F}{\restriction}a \ni \varphi_a ::= \mathrm{true} \mid \neg\varphi_a \mid \varphi_{a,1} \wedge \varphi_{a,2} \mid \mathsf{K}_a\,\varphi$ where $\varphi \in \mathscr{F}$

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$$a : P_a \parallel \vec{E}, \mathcal{K} \xrightarrow{\alpha} a : P'_a \parallel \vec{E}, \mathcal{K}' \quad \text{if } P_a, \mathcal{K} \xrightarrow{\alpha} P'_a, \mathcal{K}'$$

Symbolic Epistemic States

Focus formulæ $\Delta \subseteq \mathscr{F}$

- like in predicate abstraction

Symbolic epistemic state $\Gamma \subseteq \Delta$ such that

- Γ is Δ-closed: $\Gamma \models \varphi$ implies $\varphi \in \Gamma$ for all $\varphi \in \Delta$
- global knowledge base

Symbolic update $\Gamma \lhd^{\Delta} \alpha = \{\varphi \in \Delta \mid \Gamma \models \mathrm{wlp}(\alpha, \varphi)\}$

- weakest liberal precondition

$$\mathfrak{K} \models \mathrm{wlp}(\mathfrak{u}, \varphi) \iff \mathfrak{K} \not\models \mathit{pre}(\mathfrak{u}) \text{ or } \mathfrak{K} \lhd \mathfrak{u} \models \varphi$$
$$\mathfrak{K} \models \mathrm{wlp}(\alpha, \varphi) \iff \mathfrak{K} \models \textstyle\bigwedge_{\mathfrak{u} \in \alpha} \mathrm{wlp}(\mathfrak{u}, \varphi)$$
$$\mathcal{K} \models \mathrm{wlp}(\alpha, \varphi) \iff \mathcal{K} \lhd \alpha \models \varphi$$

Epistemic Processes and Ensembles on Symbolic States

Symbolic operational process semantics on a symbolic state $\Gamma \subseteq \Delta$

Symbolic operational ensemble semantics on a symbolic state $\Gamma \subseteq \Delta$

Processes only contain (pre-)conditions in Δ

Epistemic Processes and Ensembles on Symbolic States

Symbolic operational process semantics on a symbolic state $\Gamma \subseteq \Delta$

$$\alpha.P, \Gamma \xrightarrow{\alpha}^{\Delta} P, \Gamma \lhd^{\Delta} \alpha \quad \text{if } \Gamma \models \mathit{pre}(\alpha)$$

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$$a : P_a \parallel \vec{E}, \Gamma \xrightarrow{\alpha}^{\Delta} a : P'_a \parallel \vec{E}, \Gamma' \quad \text{if } P_a, \Gamma \xrightarrow{\alpha}^{\Delta} P'_a, \Gamma'$$

Processes only contain (pre-)conditions in Δ

Symbolic Updates: Weakest Liberal Preconditions

Given focus formulæ Δ, how can

- $\Gamma \models \mathit{pre}(\alpha)$ be decided?
- $\Gamma \lhd^{\Delta} \alpha = \{\varphi \in \Delta \mid \Gamma \models \mathrm{wlp}(\alpha, \varphi)\}$ be computed?

Weakest liberal preconditions recursively computable in $\mathscr{F}$

H. van Ditmarsch, W. van der Hoek, B. Kooi. Dynamic Epistemic Logic, 2008.

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Weakest liberal preconditions recursively computable in $\mathscr{F}$

$$
\begin{aligned}
\mathrm{wlp}(\mathfrak{u}, p) &\leftrightarrow \mathit{pre}(\mathfrak{u}) \to p \\
\mathrm{wlp}(\mathfrak{u}, \mathrm{true}) &\leftrightarrow \mathrm{true} \\
\mathrm{wlp}(\mathfrak{u}, \neg\varphi) &\leftrightarrow \mathit{pre}(\mathfrak{u}) \to \neg\mathrm{wlp}(\mathfrak{u}, \varphi) \\
\mathrm{wlp}(\mathfrak{u}, \varphi_1 \land \varphi_2) &\leftrightarrow \mathrm{wlp}(\mathfrak{u}, \varphi_1) \land \mathrm{wlp}(\mathfrak{u}, \varphi_2) \\
\mathrm{wlp}(\mathfrak{u}, \mathsf{K}_a\, \varphi) &\leftrightarrow \mathit{pre}(\mathfrak{u}) \to \textstyle\bigwedge_{q \in F(\mathfrak{u})_a} \mathsf{K}_a\, \mathrm{wlp}(\mathfrak{u} \cdot q, \varphi) \\
\mathrm{wlp}(\alpha, \varphi) &\leftrightarrow \textstyle\bigwedge_{\mathfrak{u} \in \alpha} \mathrm{wlp}(\mathfrak{u}, \varphi)
\end{aligned}
$$

H. van Ditmarsch, W. van der Hoek, B. Kooi. Dynamic Epistemic Logic, 2008.

Symbolic Updates: Δ-representability (1)

Epistemic choice action α Δ-representable if

1. there is a $\rho \in \Delta$ with $\models \mathit{pre}(\alpha) \leftrightarrow \rho$
2. for each $\varphi \in \Delta$ there is a $\rho \in \Delta$ with $\models \mathit{pre}(\alpha) \to (\mathrm{wlp}(\alpha, \varphi) \leftrightarrow \rho)$

Example $\Delta = \{\mathrm{true}, \mathsf{K}_1\, x_1, \mathsf{K}_2\, x_1, \mathsf{K}_1\, \mathsf{K}_2\, x_1\}$

$$\alpha_{x_1}^{\mathsf{k}} = \{(U_{grp}(\{2\}, \mathsf{K}_1\, \mathrm{x}_1), \mathsf{k}), (U_{grp}(\{2\}, \mathsf{K}_1\, \neg \mathrm{x}_1), \mathsf{k})\}$$

$$\mathit{pre}(\alpha_{x_1}) = \mathsf{K}_1\, x_1 = (\mathsf{K}_1\, \mathrm{x}_1) \vee (\mathsf{K}_1\, \neg \mathrm{x}_1)$$

$$\models \mathit{pre}(\alpha_{x_1}) \to (\mathrm{wlp}(\alpha_{x_1}, \begin{array}{l} \mathsf{K}_1\, x_1 \\ \mathsf{K}_2\, x_1 \\ \mathsf{K}_1\, \mathsf{K}_2\, x_1 \end{array}) \leftrightarrow \begin{array}{l} \mathrm{true} \\ \mathrm{true} \\ \mathsf{K}_1\, \mathsf{K}_2\, x_1 \end{array})$$

Symbolic Updates: Δ-representability (2)

Epistemic choice action α Δ-representable if

1. there is a $\rho \in \Delta$ with $\models \mathit{pre}(\alpha) \leftrightarrow \rho$
2. for each $\varphi \in \Delta$ there is a $\rho \in \Delta$ with $\models \mathit{pre}(\alpha) \rightarrow (\mathrm{wlp}(\alpha, \varphi) \leftrightarrow \rho)$

Example $\Delta = \{\mathrm{true}, \mathsf{K}_1\, x_1, \mathsf{K}_2\, x_1, \mathsf{K}_1\, \mathsf{K}_2\, x_1\}$

$$\alpha_{x_1}^{1,2} = \{(U_{grp}(\{1,2\}, \mathsf{K}_2\, x_1), \mathsf{k})\}$$

$$\mathit{pre}(\alpha_{x_1}^{1,2}) = \mathsf{K}_2\, x_1 = (\mathsf{K}_2\, \mathrm{x}_1) \vee (\mathsf{K}_2\, \neg \mathrm{x}_1)$$

$$\models \mathit{pre}(\alpha_{x_1}^{1,2}) \rightarrow (\mathrm{wlp}(\alpha_{x_1}^{1,2}, \mathsf{K}_1\, x_1) \leftrightarrow \mathsf{K}_1\, \mathsf{M}_2\, \neg \mathrm{x}_1 \vee \mathsf{K}_1\, \mathsf{M}_2\, \mathrm{x}_1)$$

$\mathsf{K}_1\, x_1$	$\mathsf{K}_1\, \mathsf{M}_2\, \neg \mathrm{x}_1 \vee \mathsf{K}_1\, \mathsf{M}_2\, \mathrm{x}_1$
$\mathsf{K}_2\, x_1$	true
$\mathsf{K}_1\, \mathsf{K}_2\, x_1$	true

Symbolic Updates: Δ-representability (3)

Epistemic choice action α Δ-representable if

1. there is a $\rho \in \Delta$ with $\models \mathit{pre}(\alpha) \leftrightarrow \rho$
2. for each $\varphi \in \Delta$ there is a $\rho \in \Delta$ with $\models \mathit{pre}(\alpha) \rightarrow (\mathrm{wlp}(\alpha, \varphi) \leftrightarrow \rho)$

Example $\Delta = \{\mathrm{true}, \mathsf{K}_1\, x_1, \mathsf{K}_2\, x_1, \mathsf{K}_1\, \mathsf{K}_2\, x_1, \mathsf{K}_1\, \mathsf{M}_2\, \neg \mathrm{x}_1 \vee \mathsf{K}_1\, \mathsf{M}_2\, \mathrm{x}_1\}$

$$\alpha^{\mathsf{n}}_{\mathrm{x}_1} = \{(U_{grp}(\{2\}, \mathsf{K}_1\, \mathrm{x}_1), \mathsf{n})\}$$
$$\alpha^{\mathsf{n}}_{\neg\mathrm{x}_1} = \{(U_{grp}(\{2\}, \mathsf{K}_1\, \neg\mathrm{x}_1), \mathsf{n})\}$$

$$\mathit{pre}(\alpha^{\mathsf{n}}_{\mathrm{x}_1}) = \mathrm{true} = \mathit{pre}(\alpha^{\mathsf{n}}_{\neg\mathrm{x}_1})$$

$$\models \mathrm{wlp}(\alpha^{\mathsf{n}}_{\mathrm{x}_1}, \mathsf{K}_1\, \mathsf{M}_2\, \neg\mathrm{x}_1 \vee \mathsf{K}_1\, \mathsf{M}_2\, \mathrm{x}_1) \leftrightarrow$$
$$(\mathsf{M}_1\, \neg\mathrm{x}_1 \wedge \mathsf{K}_1\, \mathsf{M}_2\, \neg\mathrm{x}_1) \vee \mathsf{K}_1\, \mathsf{M}_2\, \mathrm{x}_1$$

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`https://github.com/AlexanderKnapp/epistemic.git`

Syntactic Symbolic Updates

If $\Gamma \subseteq \Delta$ is Δ-closed, then also $\Gamma \lhd^{\Delta} \alpha$ is Δ-closed.

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Syntactic variant of symbolic updates for Δ-representable α

$$\Gamma \ll^{\Delta} \alpha = \{\varphi \in \Delta \mid \text{ex. } \rho \in \Gamma \text{ s.t. } \models \mathit{pre}(\alpha) \rightarrow (\mathrm{wlp}(\alpha, \varphi) \leftrightarrow \rho)\}$$

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Lemma Let $\Gamma \models \mathit{pre}(\alpha)$ for Δ-representable α. Then $\Gamma \lll^{\Delta} \alpha = \Gamma \lhd^{\Delta} \alpha$.

Equivalence of Concrete and Symbolic Semantics

$\mathcal{K} \equiv^{\Delta} \Gamma$ iff $\mathcal{K} \models \varphi \iff \Gamma \models \varphi$ for all $\varphi \in \Delta$

Theorem Let α be Δ-representable. Let $\mathcal{K} \equiv^{\Delta} \Gamma$.

1. If $\vec{E}, \mathcal{K} \xrightarrow{\alpha} \vec{E}', \mathcal{K}'$, then $\vec{E}, \Gamma \xrightarrow{\alpha}{}^{\Delta} \vec{E}', \Gamma'$ with $\mathcal{K}' \equiv^{\Delta} \Gamma'$.
2. If $\vec{E}, \Gamma \xrightarrow{\alpha}{}^{\Delta} \vec{E}', \Gamma'$, then $\vec{E}, \mathcal{K} \xrightarrow{\alpha} \vec{E}', \mathcal{K}'$ with $\mathcal{K}' \equiv^{\Delta} \Gamma'$.

Distributed Symbolic Epistemic States

Distributed focus formulæ $\vec{\nabla} = (\nabla_a)_{a \in A}$

- $\vec{\Phi} = (\Phi_a)_{a \in A}$ distributed if $\bigcup \vec{\Phi} \cap \mathscr{F}{\restriction}a = \Phi_a$ for all $a \in A$

Distributed symbolic epistemic state $\vec{\Lambda} = (\Lambda_a \subseteq \nabla_a)_{a \in A}$ such that

- $\bigcup \vec{\Lambda}$ is $\bigcup \vec{\nabla}$-closed
- agent-wise local knowledge bases

Symbolic update $\vec{\Lambda} \lhd^{\vec{\nabla}} \alpha = (\Lambda_a \lhd^{\nabla_a} \alpha)_{a \in A}$

$$\Lambda_a \lhd^{\nabla_a} \alpha = \{\varphi \in \nabla_a \mid \Lambda_a \models \mathrm{wlp}(\alpha, \varphi)\}$$

- also with syntactic variant

Distributed Epistemic Ensembles

Distributed operational ensemble semantics on a distributed state $\vec{\Lambda} \subseteq \vec{\nabla}$

- initiator i
- at most affected group C
  - $\Lambda_a \lhd^{\nabla_a} \alpha = \Lambda_a$ for all $a \in A \setminus C$

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$$i : (P_i, \Lambda_i) \parallel (a : (P_a, \Lambda_a))_{a \in C} \parallel \vec{D} \xrightarrow{\alpha}^{\vec{\nabla}} i : (P'_i, \Lambda'_i) \parallel (a : (P_a, \Lambda'_a))_{a \in C} \parallel \vec{D}$$

$$\text{if } P_i, \Lambda_i \xrightarrow{\alpha} P'_i, \Lambda'_i \text{ and } (\Lambda_a)_{a \in C} \lhd^{(\nabla_a)_{a \in C}} \alpha = (\Lambda'_a)_{a \in C}$$

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Distributed Updates: $\vec{\nabla}$-representability

Given distributed focus formulæ $\vec{\nabla} = (\nabla_a)_{a \in A}$, how can

- $\Lambda_a \models \mathit{pre}(\alpha)$ be decided?
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- the "affected group" of an action be determined?

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1. there is a $\rho_a \in \nabla_a$ with $\models \mathit{pre}(\alpha) \leftrightarrow \rho_a$
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Example $\mathsf{K}_3\, x_1 \in \nabla_3$

$$\models \mathsf{K}_2\, x_1 \to (\mathrm{wlp}(\alpha^{1,2}_{x_1}, \mathsf{K}_3\, x_1) \leftrightarrow \mathsf{K}_3\, x_1)$$

Equivalence of Distributed and Symbolic Semantics

Theorem Let $(a : (P_a, \Lambda_a))_{a \in A}$ be $\vec{\nabla}$-distributed and let α be $\vec{\nabla}$-representable.

$$(a : (P_a, \Lambda_a))_{a \in A} \xrightarrow{\alpha}^{\vec{\nabla}} (a : (P'_a, \Lambda'_a))_{a \in A} \iff$$
$$(a : P_a)_{a \in A}, \bigcup \vec{\Lambda} \xrightarrow{\alpha}^{\bigcup \vec{\nabla}} (a : P'_a)_{a \in A}, \bigcup \vec{\Lambda}'$$

Conclusions and Future Work

Executing epistemic ensembles

- concretely, symbolically, and distributed
- using techniques from symbolic execution and predicate abstraction

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Executing epistemic ensembles

- concretely, symbolically, and distributed
- using techniques from symbolic execution and predicate abstraction

- When do finite abstractions exist?
- How to handle infinite sets of focus formulæ?
- relation to knowledge-based programs
  (R. Fagin, J. Y. Halpern, Y. Moses, M. Y. Vardi)
- applications to distributed communication protocols
  (M. Herlihy, D. Kozlov, S. Rajsbaum)