Some notes on teaching quantum programming

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A Specialization Path in UM MSc in Informatics Eng

Context

- 20 ECTS (out of 60) distributed into 4 courses:

Sem 1	Quantum computation computational model algorithms	Platforms architectural paradigms simulation & experiments
Sem 2	Quantum Logic logical & algebraic foundations	Quantum ML applications to ML hybrid algorithms

- possible combinations: Formal Methods, Distributed System, Cryptography, Machine Learning, Software Engineering, etc.

Challenges

Qantum is trendy ... but weird ... still at a proof-of-concept stage ...

Relevance

Two main intelectual achievements of the 20th century met

- Computer Science and Information theory progressed by abstracting from the physical reality. This was the key of its success to an extent that its origin was almost forgotten.
- On the other hand, quantum mechanics ubiquitously underlies ICT devices at the implementation level, but had no influence on the computational model itself ... until now!

Proof-of-concept implementations available ...

— and some pressure from industry to include in regular curricula

The Quantum Computing course

Background: undergrad complex vector spaces and basic linear algebra.

Syllabus

- Quantum effects as computational resources: superposition, interference, entanglement
- The computational model: The representation, evolution, composition and measurement postulates
- The golden patterns
- Quantum algorithms
  - based on phase amplification
  - based on the quantum Fourier transform
- Quantum programming in PennyLane

Difficulty: quantum phenomena as resources

Probabilistic systems, probabilistic computation

In any case,

Computation is always a physical process

That's our *motto*!

In several cases, the language of probability theory can describe the actual physical evolution of a system, i.e.

- Physics identify the system's structure and assigns numerical probabilities to elementary transition steps.
- Probability theory, i.e. the Kolmogorov axioms, ensure mathematical consistency and helps in calculating probabilities along paths of evolution.

Extracts from the course slides

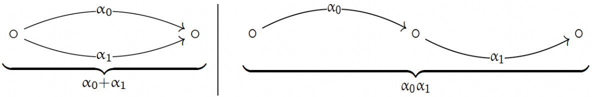
Quantum systems, quantum computation

Many common quantum phenomena, however, cannot be described this way, but are accommodated by a modified 'probability theory':

Transitions are labelled by complex numbers, called their amplitudes, whose norms squared are interpreted as transition probabilities through

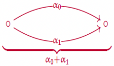
$$\text{Born's rule} \quad p = |\alpha|^2$$

(for Max Born, 1882-1970)



Extracts from the course slides

Quantum systems, quantum computation

Let's compute the total probability in 

$$\begin{aligned}
p &= |\alpha_0 + \alpha_1|^2 \\
&= \overline{(\alpha_0 + \alpha_1)}(\alpha_0 + \alpha_1) \\
&= (\overline{\alpha_0} + \overline{\alpha_1})(\alpha_0 + \alpha_1) \\
&= |\alpha_0|^2 + |\alpha_1|^2 + \overline{\alpha_0}\alpha_1 + \alpha_0\overline{\alpha_1} \\
&= p_0 + p_1 + |\alpha_0||\alpha_1| \left(e^{i(\varphi_1 - \varphi_0)} + e^{-i(\varphi_1 - \varphi_0)} \right) \\
&= p_0 + p_1 + \underbrace{2\sqrt{p_0 p_1}\cos(\varphi_1 - \varphi_0)}_{\text{interference}}
\end{aligned}$$

(expressing α_j in polar form $|\alpha_j|e^{i\theta_j}$
and resorting to $e^{i\theta} + e^{-i\theta} = 2\cos\theta$)

Extracts from the course slides

Quantum systems, quantum computation

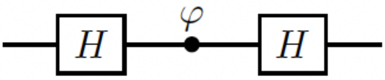
$$p = p_0 + p_1 + \underbrace{2\sqrt{p_0 p_1}\cos(\varphi_1 - \varphi_0)}_{\text{interference}}$$

- The total probability is the sum of the probabilities of the individual transitions modified by the interference term.
- Depending on term $\varphi_1 - \varphi_0$ the interference can be either negative or positive.
- The important quantity is the relative phase $\varphi_1 - \varphi_0$ rather than individual φ_0, φ_1
- If the system's evolution depends only on that difference then the system must have, somehow, experienced both paths.

Difficulty: algorithmic patterns

The whole course builds on two simple patterns, revisiting them along algorithmic development

- The interference pattern



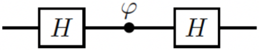
- and its combination with entanglement as introduced through controlled (spy) operators

... maybe instructive to look again at the course slides as this is the cornerstone of the whole approach:

Extracts from the course slides

Superposition & interference

My first quantum circuit



- A wire represents a two-dimensional state (a qubit)
- Three gates describing quantum operations:

$$\underbrace{H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}}_{\text{Hadamard gate}} \quad \text{and} \quad \underbrace{P_\varphi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix}}_{\text{Phase shift gate}}$$

A simple matrix multiplication yields

$$A = HP_\varphi H = \begin{bmatrix} \cos\frac{\varphi}{2} & -i\sin\frac{\varphi}{2} \\ -i\sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{bmatrix}$$

Extracts from the course slides

Superposition & interference

My first quantum circuit

which, expressed in a functional way, gives

$$A|0\rangle = \cos\frac{\varphi}{2}|0\rangle + -i\sin\frac{\varphi}{2}|1\rangle$$

$$A|1\rangle = -i\sin\frac{\varphi}{2}|0\rangle + \cos\frac{\varphi}{2}|1\rangle$$

as read from

$$A = HP_{\varphi}H = \begin{bmatrix} \cos\frac{\varphi}{2} & -i\sin\frac{\varphi}{2} \\ -i\sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} \end{bmatrix}$$

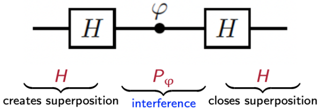
Clearly, for $\varphi = 0$, i.e. in the absence of any phase shift, $A|0\rangle = |0\rangle$ and $A|1\rangle = |1\rangle$, leading to conclude that

$$HH = Id$$

Extracts from the course slides

Superposition & interference

The golden pattern



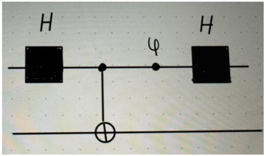
- *H* creates/closes a uniform superposition: it is the source of a natural parallelism,
- but the crucial role in controlling interference is located in P_φ.

Extracts from the course slides

Entanglement comes in

Entanglement and Interference

Let's combine the interference and the entanglement patterns:



and apply it to $|00\rangle$:

$$(H \otimes I)(P_\varphi \otimes I)\,CNOT\,(H \otimes I)\,|00\rangle$$

Extracts from the course slides

Entanglement comes in

Entanglement and Interference

$$(H \otimes I)(P_\varphi \otimes I)\,CNOT\,(H \otimes I)\,|00\rangle$$

- Before P_φ the system is in the first Bell state; after it becomes

$$\frac{1}{\sqrt{2}}(|00\rangle + e^{i\phi}|11\rangle)$$

- The second H closes superposition yielding

$$\frac{1}{2}(|00\rangle + |10\rangle + e^{i\varphi}|01\rangle - e^{i\varphi}|11\rangle) = |0\rangle\left(\frac{|0\rangle + e^{i\varphi}|1\rangle}{2}\right) + |1\rangle\left(\frac{|0\rangle - e^{i\varphi}|1\rangle}{2}\right)$$

Extracts from the course slides

Entanglement comes in

Entanglement and Interference

An easy calculation shows that the probabilities of measuring 0 or 1 are $\frac{1}{2}$ in each case.

Interference (via P_φ) was cancelled by entanglement: Indeed the second qubit learnt the first one, becoming aware of which path in the superposition was taken.

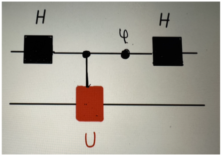
The *CNOT* gate, the spy gate, destroyed interference

Extracts from the course slides

The general pattern

Entanglement and Interference

Let's go general: replacing *CNOT* by a controlled C_U for an arbitrary unitarian U:



$$(H \otimes I)(P_\varphi \otimes I)C_U(H \otimes I)\,|00\rangle$$

Extracts from the course slides

The general pattern

Entanglement and Interference

$$C_U = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U$$

- If the control qubit is $|0\rangle$, the target qubit remains $|0\rangle$
- if not it is rotated from $|0\rangle$ to $U|0\rangle$

As before, if $U = CNOT$ the rotation is π: the new state becomes orthogonal to the previous one.
The second qubit acquired full knowledge about the first one.

In general, however, the acquired knowledge may be imperfect:

$$\boxed{\langle 0|U|0\rangle = ve^{-i\alpha}}$$

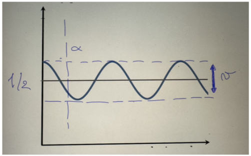
recall: the inner product measures the similarity between states

Extracts from the course slides

The general pattern

Entanglement and Interference

$$\langle 0|U|0\rangle = ve^{-i\alpha}$$



Special cases

$\langle 0|X|0\rangle = 0$ Interference completely suppressed: the curve becomes flat at $\frac{1}{2}$.

$\langle 0|Id|0\rangle = 1$ Full interference: probabilities oscillate as $\cos\frac{\varphi}{2}$.

The Quantum Logic course

Syllabus

- Module 1: A brief introduction to category theory.
  - What is a category and why we care.
  - Functors and natural transformations.
  - Basic constructions in a category: duality and universality.
  - Monads and adjunctions.
- Module 2: A diagrammatical approach to quantum processes
  - Monoidal categories and string diagrams.
  - Computational interpretation of quantum mechanics. Associated categorical structures: monoidal (composition), compact closed (entanglement), adjunctions (internal product), biproduts (non deterministic branching).
  - Linear and quantum processes.
  - A hands-on introduction to the ZX-calculus and PyZX.
  - Examples and case studies.

The Quantum Logic course

Remarks

- Focus: In spite of stressing foundational stuff, what catches students is ZX and PyZX
- Consolidation project: Analysis in ZX of hybrid algorithms from the Quantum ML twin course
- References: [Coecke & Kissinger, 17], [Heunen & Vicary, 19], [Kissinger & Wetering, 24]
- Failed alternative syllabus:
  - revisiting the Curry-Howard-Lambek correspondence
  - and (some variants of the) quantum λ-calculus

Questions

Current methods and tools are still highly fragmentary and fundamentally 'low-level'.

- Reasoning directly with quantum gates sweeps under the carpet all key ingredients of a mature software engineering discipline: compositionality, abstraction, refinement, high-order and property-enforcing type schemes.
- Could we reframe C1 with the diagrammatic language of C2?
- Or, more conventionally, define a programming language, its operational semantics and an associated dynamic logic?
- How to extend whatever approach to (the increasingly relevant) hybrid programs?
- How to incorporate classical, macroscopic noise into the picture, in an effective, not implicit way.?

Questions

The conceptualisation of quantum computing predated its technological realisation as, in the 1930's, Turing machines anticipated digital computers.

It seems history is repeating itself. Differently, however, from what happened before, we may have the chance to get theory in place before technologies emerge and popularise.

... and teach the subject accordingly