# Symbolic Runtime Verification for Monitoring under Uncertainties and Assumptions

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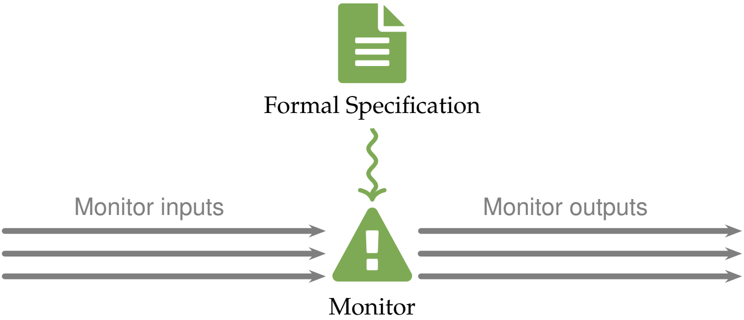
# General Setting: Runtime Verification



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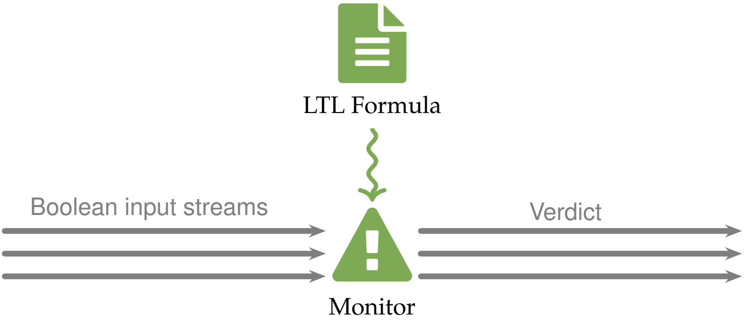
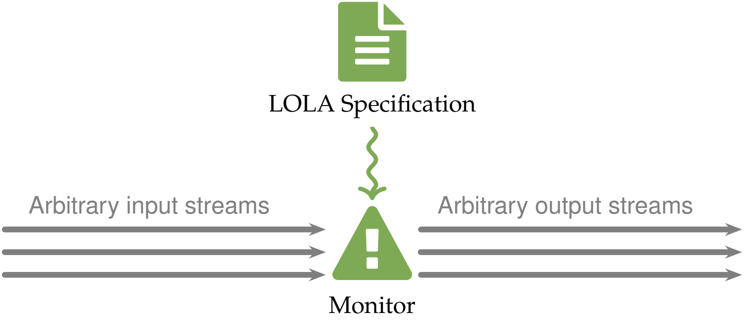
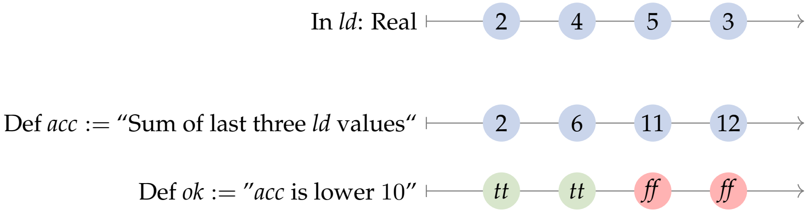


# General Setting: Stream Runtime Verification



# Stream Runtime Verification: Example



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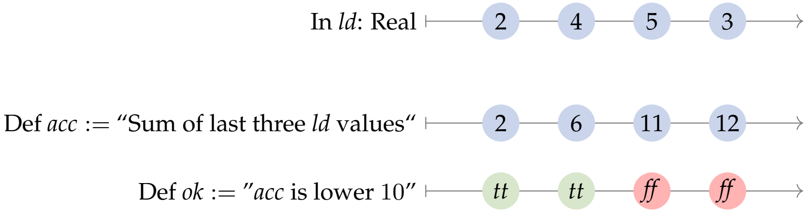


Three basic LOLA stream expressions:

- Constant streams e.g. $10$
- Offset operators $s[o|c]$
  $\Rightarrow$: We restrict our self to the past fragment here (i.e. $o \leq 0$)
- Function applications e.g. $a[now] + b[now]$

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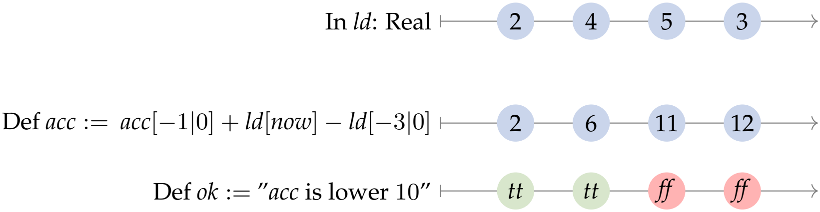


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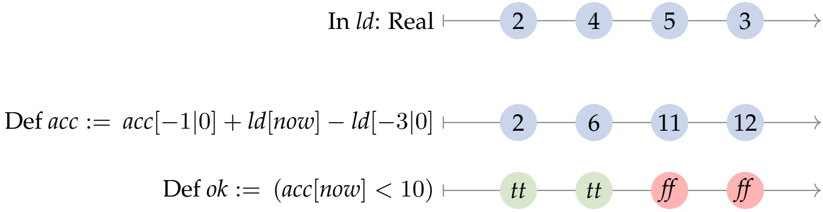


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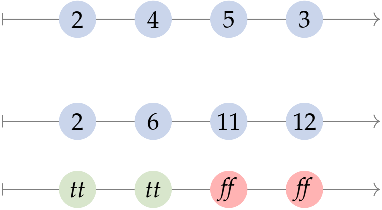
# Research Question: Uncertainties & Assumptions

In *ld*: Real

Def $acc := acc[-1|0] + ld[now] - ld[-3|0]$

Def $ok := (acc[now] < 10)$



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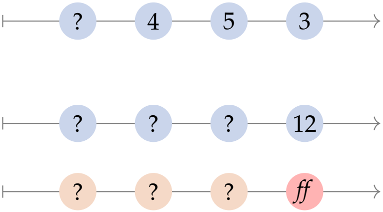


**Uncertainty:** What to do when some events are (partially) unknown?

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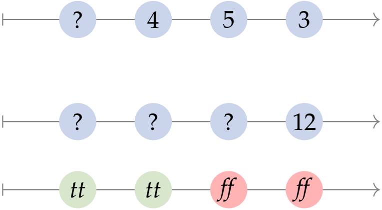


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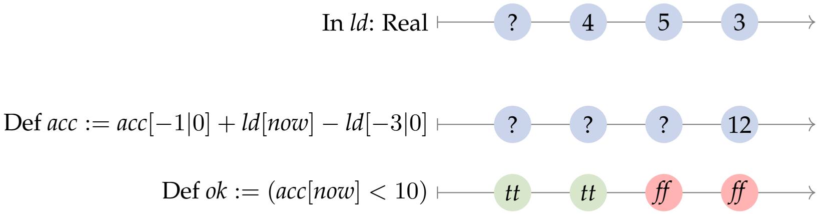


**Assumptions:** How to use additional information about the system?

E.g. The value of every input is between 1 and 5

$\Rightarrow$ In general very powerful!

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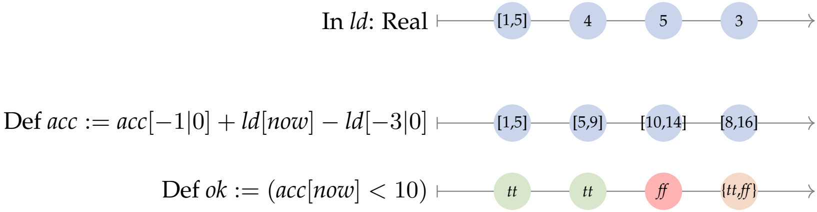

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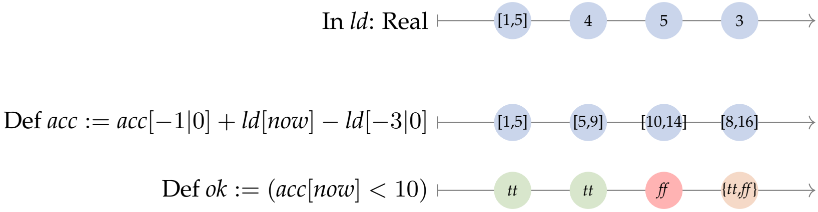


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| | | | | |
|---|---|---|---|---|
| In *ld*: Real | [1,5] | 4 | 5 | 3 |
| Def $acc := acc[-1\|0] + ld[now] - ld[-3\|0]$ | [1,5] | [5,9] | [10,14] | [8,16] |
| Def $ok := (acc[now] < 10)$ | *tt* | *tt* | *ff* | {*tt*,*ff*} |

- Approach is sound, but not perfect.
- Handling of complex assumptions not clear.

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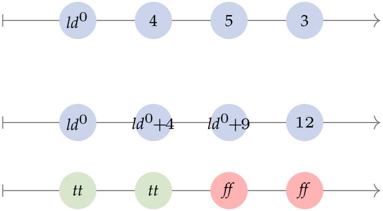


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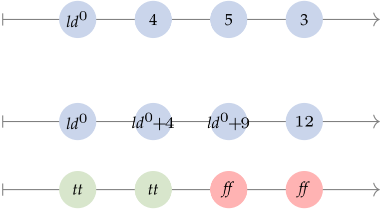


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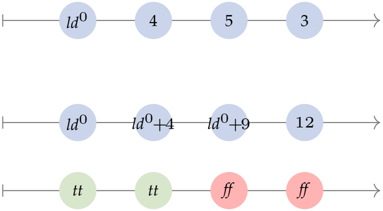


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- Approach in principle perfect.
- Assumptions up to $t$ can be added as propositions to constraint set.

# Symbolic SRV: Monitoring Algorithm

**Algorithm:** Symbolic Monitoring Algorithm for LOLA specification $\varphi$

$t \leftarrow 0$ and $E \leftarrow \emptyset$;
**while** $t \in \mathbb{T}$ **do**
    Read Input$^t$;
    $E \leftarrow E \cup [\![\varphi]\!]^t_{sym}$ ;
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Assumption knowledge up to timestamp $t$ can also be included.

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    Output;
    Prune;
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$\Rightarrow$ **No monitoring with trace-length-independent resources!**

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$$\begin{array}{rcl} a & := & a[-1|\mathit{ff}] \oplus x[\mathit{now}] \\ b & := & b[-1|\mathit{tt}] \oplus x[\mathit{now}] \\ \mathit{ok} & := & a[\mathit{now}] \oplus b[\mathit{now}] \quad // = \mathit{true} \end{array}$$

| $t$ | 0 | 1 | 2 | 3 | … |
|---|---|---|---|---|---|
| $a^t$ | $x^0$ | $x^0 \oplus x^1$ | $x^0 \oplus x^1 \oplus x^2$ | $x^0 \oplus x^1 \oplus x^2 \oplus x^3$ | … |
| $b^t$ | $\neg x^0$ | $\neg x^0 \oplus x^1$ | $\neg x^0 \oplus x^1 \oplus x^2$ | $\neg x^0 \oplus x^1 \oplus x^2 \oplus x^3$ | … |
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| $ok^t$ | $tt$ | $tt$ | $tt$ | $tt$ | ... |

**Observation:** Growing formulas in steps 1, 2, 3 describe only two possible vectors: $(a^3, b^3, ok^3) \in \{(ff, tt, tt), (tt, ff, tt)\}$.

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For the **Boolean fragment** trace-length independent symbolic monitoring is always possible!

# Pruning of symbolic formulas

**Second case: Linear Algebra LOLA Fragment**
(Only Real streams/assumptions of form
$s = c_1 \cdot s_1[o_1, d_1] + \cdots + c_n \cdot s_n[o_n, d_n]$)

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$$\begin{aligned} acc_a &:= acc_a[-1|0] + ld_a[now] \\ acc_b &:= acc_b[-1|0] + ld_b[now] \\ total &:= total[-1|0] + \tfrac{1}{2}(ld_a[now] + ld_b[now]) \end{aligned}$$

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$$\begin{pmatrix} acc_a^1 \\ acc_b^1 \\ total^1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} * \begin{pmatrix} ld_a^0 \\ ld_b^0 \\ ld_a^1 \\ ld_b^1 \end{pmatrix}$$

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$$\begin{pmatrix} \mathit{acc}_a^1 \\ \mathit{acc}_b^1 \\ \mathit{total}^1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} * \begin{pmatrix} u^1 \\ v^1 \end{pmatrix}$$

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**A perfect pruning is not possible here!**

$$\begin{array}{lcl} x & := & x[-1|0] + i[now] \\ y & := & 2 * y[-1|0] + i[now] \end{array}$$

$$\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix} * \begin{pmatrix} i^0 \\ i^1 \\ i^2 \end{pmatrix}$$

Assumption: $0 \leq i[now] \leq 1$

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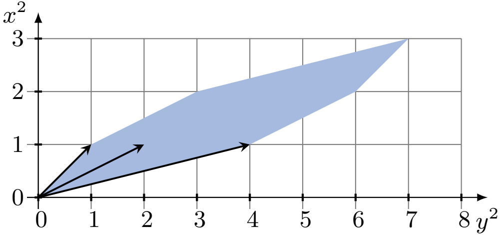

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**Sound, but imperfect (over-approximating) pruning strategy:**

- Perform pruning strategies for Boolean and Real streams separately.
- Calculate (over approximating) bounds for new Real variables (e.g. with Linear Arithmetic Optimizer).
- Add calculated bounds as constraints.

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$\Rightarrow$ Leads to imperfect over-approximation of exact polyhedron.

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But: Over-approximation possible

# Addition: Quantifier-Elimination

Quantifier elimination can be used to get a perfect pruning strategy.

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**But:** QE is usually perfect, but not constant, and does not exist for every logic.

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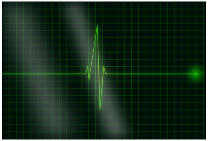

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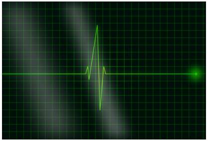

⇒ Introduced different kinds of uncertainty into the traces and added assumptions.

Compared to interval approach from previous publication.

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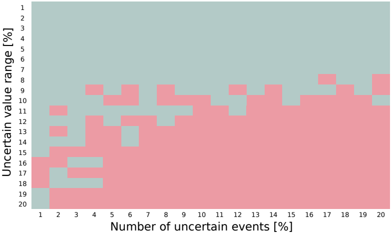

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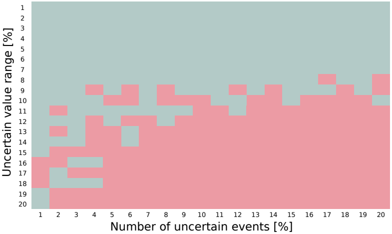


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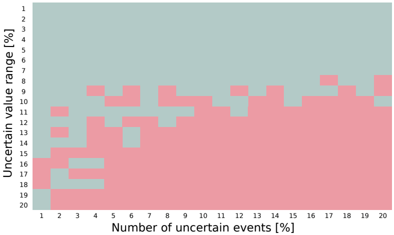


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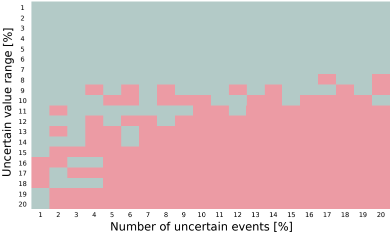


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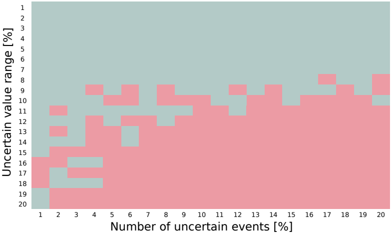


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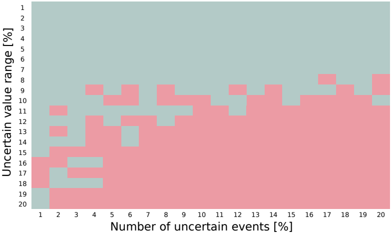


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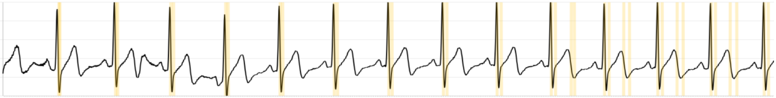

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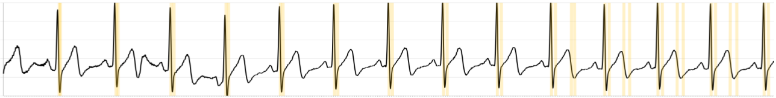

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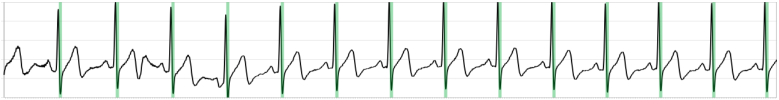
Symbolic approach with assumption (two heart peaks cannot be too close) [20% uncertain values]:



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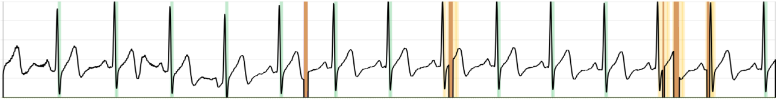

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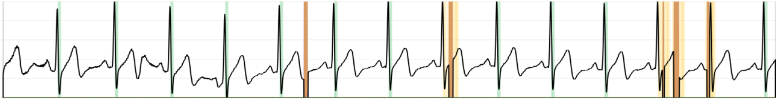

**Drawback: Symbolic approach (with assumptions) has sig. increased runtime: 25-100 ms/event**

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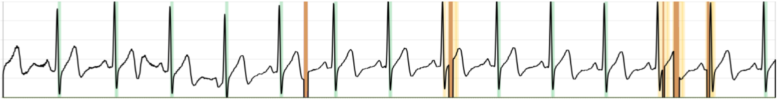

**Research question:** How to use solver more efficiently for regular problem

# Conclusion & Future Work

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**Future Work**

- Support future offsets (and future assumptions)
- Investigate further LOLA fragments
- Improve implementation