# Symbolic Analysis and Parameter Synthesis for Parametric Timed Automata and Petri Nets using Maude with SMT Solving

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Based on joint work with Carlos Olarte, Jaime Arias, Laure Petrucci (Sorbonne Paris-Nord); Kyungmin Bae (POSTECH); and Fredrik Rømming (Cambridge)

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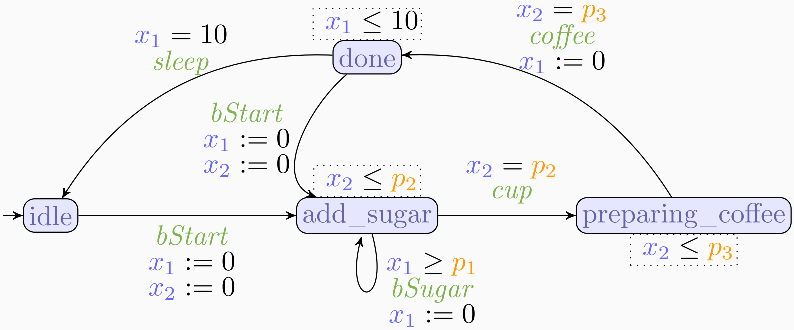
## Content

# Background

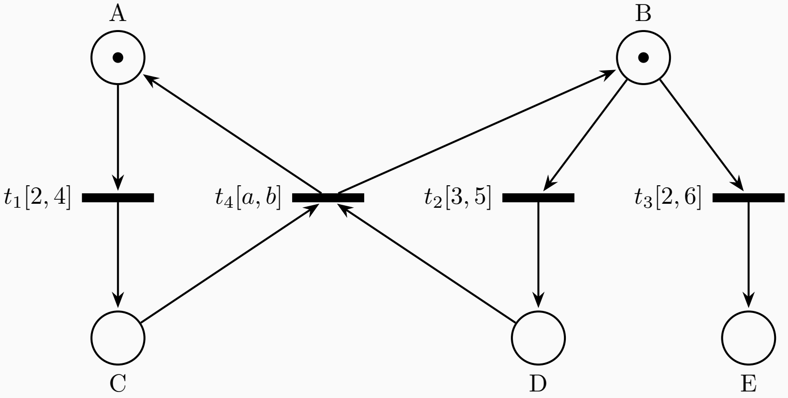
# Context: Formal Models for Real-Time Systems

- Timed automata



# Context: Formal Models for Real-Time Systems

- Timed automata
- Time(d) Petri nets



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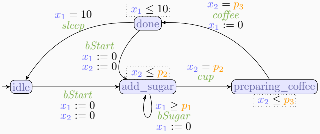


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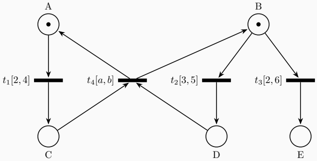


- Limited expressiveness/modeling convenience
    - data types; unbounded data structures; fixed communication; ...

\+ many properties decidable

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  - algebraic specification!

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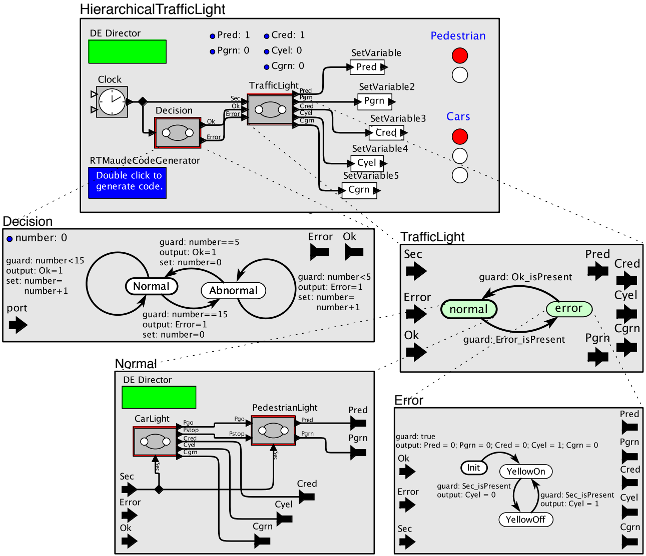
## Context: Real-Time Maude (Ölveczky and Meseguer)

- Expressiveness and modeling convenience
  - algebraic specification!
- User-defined data types; flexible communication; unbounded data structures; dynamic object/message creation/deletion; hierarchical states; ...
- Found unknown flaws in
  - scheduling with reuse of unused budgets (unbounded queues)
  - wireless sensor algorithms (communication, areas, angles)
  - MANET protocol (mobility + communication delay)
  - 50-page multicast protocol (laaarge functions; communication)
  - cloud-based transaction systems
  - cars in Japan
  - avionics systems (complex dynamics; LTL model checking)
  - ...

## Ptolemy II DE Models:



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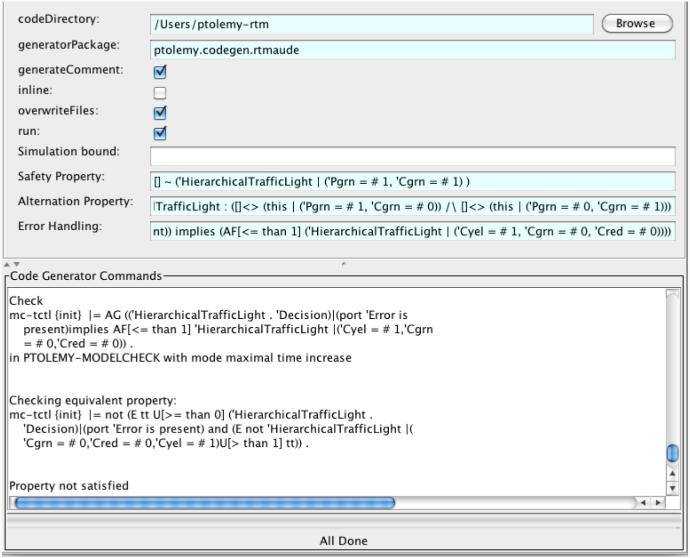
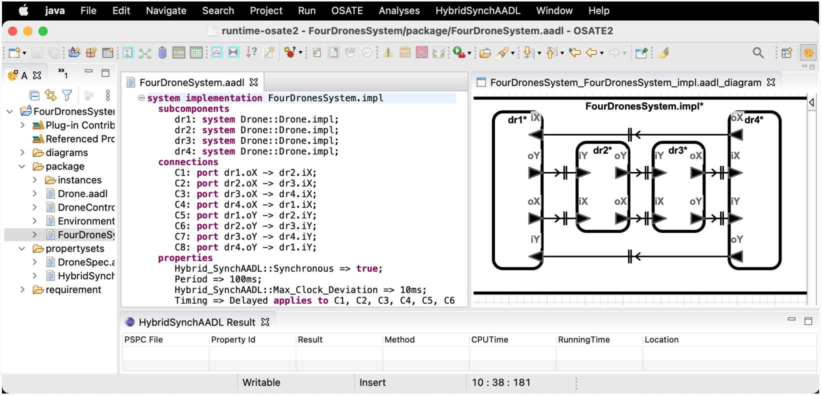


## (Synchronous) AADL:



# Semantics and Analysis for Modeling Languages

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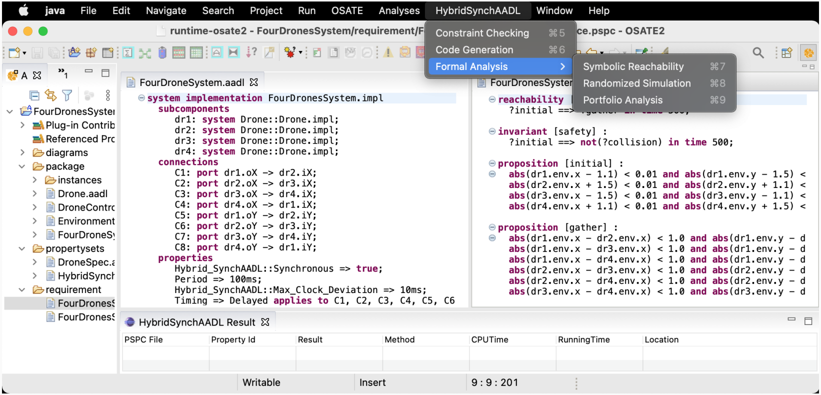

## Rewriting Logic and Maude (Meseguer)

### Rewriting Logic

- static parts: algebraic equational specification
- dynamic parts: rewrite rules

```
crl [l] :  t => u if cond .
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## Maude: Language and Analysis for Rewriting Logic

- distributed systems in OO style
- simulation
- (explicit-state) search, LTL model checking, ...
- meta-programming
- unification and narrowing

# Maude with SMT Solving

Recent connection to SMT solvers: Yices2, CVC4, Z3

```
check t .

smt-search  t =>* u such that ... . --- undocumented!

op metaCheck : Module Term ~> Bool [special (...)] .
                  --- meta-level
```

# Maude with SMT Solving "in Practice"

- Topmost rules (grab whole state)
- Symbolic "terms/states"

$$\Phi(x, y, ...) \parallel t(x, y, ...)$$

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- Symbolic "terms/states"

$$\Phi(x, y, ...) \ || \ t(x, y, ...)$$

- Rule $l : q \longrightarrow r$ **if** $\psi$ turned into rule

$$l : \ \texttt{PHI} \ || \ q \ \longrightarrow \ (\texttt{PHI} \ \textit{and} \ \psi) \ || \ r \ \ \textbf{if} \ \textit{smtCheck}(\texttt{PHI} \ \textit{and} \ \psi)$$

# Real-Time Maude (Ölveczky and Meseguer)

### Real-Time Rewrite Theories

- equational specification for data types
- rewrite rules for instantaneous change
- tick rules

  `crl [l] : {t} => {u} in time τ if ...`

  model time advance

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## Real-Time Maude

- explicit-state simulation, reachability analysis, ...
- unbounded and time-bounded analysis
- timed CTL model checking

## Explicit-State Analysis in Real-Time Maude

- Tick rules often

  crl [*tick*] : {*t*} => {*f*(*t*, *x*)} in time *x* if *x* <= *mte*(*t*) .

  - *x* new variable
  - non-executable

# Explicit-State Analysis in Real-Time Maude

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  crl [tick] :  {t} => {f(t,x)} in time x  if x <= mte(t) .
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  - all times not visited
  - analysis unsound/incomplete
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Real-Time Maude: expressive, but "unsound"

# Goal

# Current Research: Symbolic Real-Time Maude

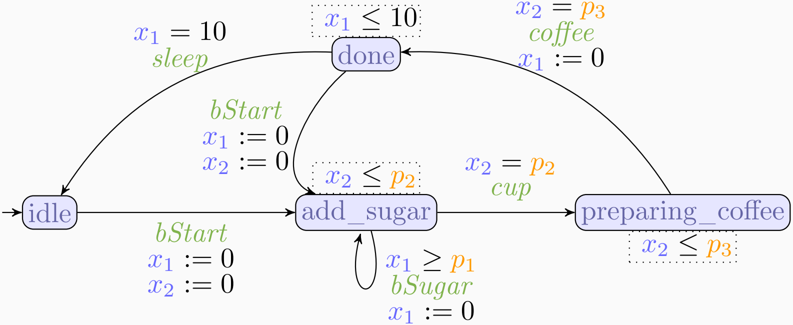
**Research Goals**

- Sound and complete analysis methods for systems beyond timed automata, ...
- Efficient rewriting-with-SMT methods for real-time systems

- Time increase encoded symbolically

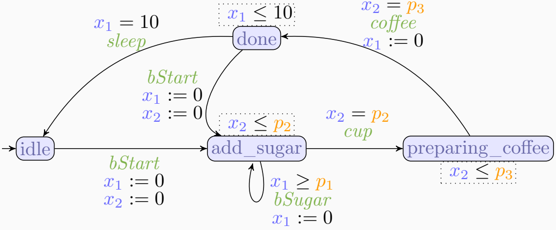
# Maude-with-SMT for Parametric Timed Automata

# Parametric Timed Automata (PTA)




Imitator: state-of-the-art PTA parameter synthesis tool (É. André)

# Representing PTA in Maude



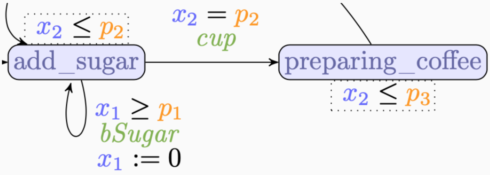

```
mod PTA-COFFEE is pr REAL .
 sorts State NState DState Location .
 subsorts NState DState < State .
 vars X1 X2 P1 P2 P3 T  : Real .

 ops idle addSugar preparingCoffee done : -> Location .

 op <_:_;_> <_;_;_> : Location Real Real Real Real Real -> DState .

 op [_:_;_] <_;_;_> : Location Real ... -> NState .
```

## Representing PTA in Maude (cont.)




```
crl [addSugar-cup] :
    < addSugar : X1 ; X2 >          < P1 ; P2 ; P3 >
   =>
    [ preparingCoffee : X1 ; X2 ]  < P1 ; P2 ; P3 >
  if (X2 === P2 and X2 <= P3) = true .

crl [preparingCoffee-tick] :
    [ preparingCoffee : X1 ; X2]            < P1 ; P2 ; P3>
   =>
    < preparingCoffee : X1 + T ; X2 + T >  < P1 ; P2 ; P3 >
      if (X2 + T <= P3 and T >= 0/1) = true [nonexec] .
```

## Analysis with Maude's `smt-search`

Is there a parameter valuation, from initial state with same clock values, such that we reach state done?

```
smt-search  [ idle : X1  ; X2  ] < P1 ; P2 ; P3 >  =>*
            < done : X1' ; X2' > < P1 ; P2 ; P3 >
    such that (X1 === X2 and X1 >= 0/1 and P1 >= 0/1
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```

```
Solution 1
state: < done : #3-T ; #1-T + #2-T + #3-T > <P1 ; P2 ; P3>

where X1 === X2 and X1 >= 0/1 and P1 >= 0/1 and P2 >= 0/1 and
  ... and #1-T:Real + #2-T:Real === P3 and
  ... and #3-T:Real <= 10/1 and #1-T:Real + #2-T:Real === P3
```

At least `X2' >= P3` ...

# Reachability

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- stop when symbolic state subsumed by earlier symbolic state
- $\phi_u \;||\; t_u \;\sqsubseteq\; \phi_v \;||\; t_v$
  iff exists $\sigma$ such that $t_u = t_v\sigma$ and $\phi_u \implies \phi_v\sigma$

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  - Maude's meta-level (`metaMatch` and `metaCheck`)

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**Theorem**

*`reachability` terminates when parametric zone graph of PTA finite*

- *+ soundness/completeness results*

# Analysis and Parameter Synthesis

- $EF\phi$-synthesis and $AG\neg\phi$-synthesis
- $\phi$ involves locations and clocks
- parameters: $\exists X.\varphi$ of resulting constraint $\varphi$
  - quantifier elimination (Z3)
  - done reachable when $p_2 \leq p_3$

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- Analysis with Maude strategies

**Example**

Can we reach done when some sugar required? (If $p_1 \leq p_2 \leq p_3$)

# Comparied to Imitator

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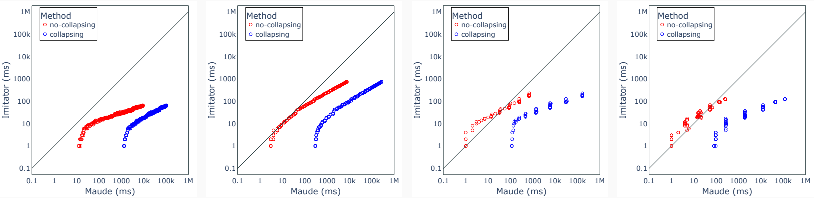
- Benchmarking on PTA library




Unfair comparison!

# PTA in Maude-SMT

- Promising
- New `reachability` method
- Parameter synthesis
- Very limited model
  - atomic states
  - no equations
  - linear constraints

## Parametric Interval Time Petri Nets with Inhibitor Arcs (PITPNs)

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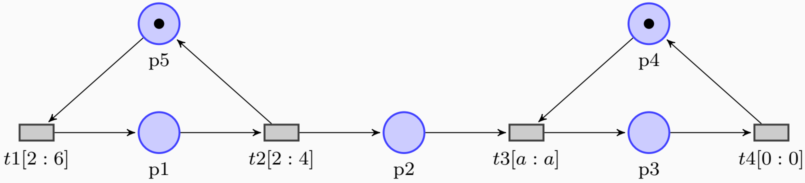


- Unbounded states
- "User-defined" functions
- "Interpreter"

# Representing Nets in Maude

Markings represented `p |-> 2 ; q |-> 3` instead of `p p q q q`

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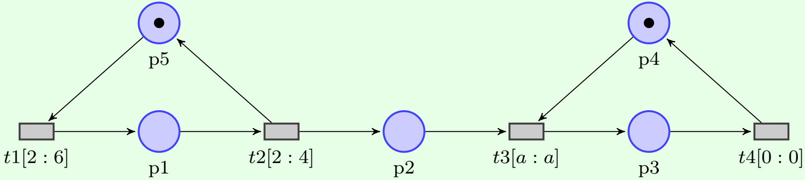


```
"t1" :  "p5" |-> 1  -->  "p1" |-> 1  in [2 : 6] ;
"t2" :  "p1" |-> 1  -->  "p2" |-> 1 ; "p5" |-> 1  in [2 : 4] ;
"t3" :  "p2" |-> 1 ; "p4" |-> 1  -->  "p3" |-> 1  in [a : a] ;
"t4" :  "p3" |-> 1  -->  "p4" |-> 1  in [0 : 0]
```

## Symbolic Rewrite Semantics: Advance Time

```
var T : Real .

crl [tick] :  tickOk    : M : CLOCKS     : NET
          =>  tickNotOk : M : increaseClocks(M, CLOCKS, NET, T) : NET
if (T >= 0/1 and mte(M, CLOCKS, NET, T)) = true .


op mte : Marking ClockValues Net Real -> Boolean .
eq mte(M, empty, NET, T) = true .
eq mte(M, (L -> R1) ; CLOCKS, (L : PRE --> ... in [T1 : inf]) ; NET, T)
 = mte(M, CLOCKS, NET, T) .
eq mte(M, (L -> R1) ; CLOCKS, (L : PRE --> ... in [T1 : T2])  ; NET, T)
 = (active(M, L : ...) ? T <= T2 - R1 : true)
   and mte(M, CLOCKS, NET, T) .
```

# Symbolic Rewrite Semantics: Applying Transition

```
crl [applyTransition] :
    TS : M : ((L -> T) ; CLOCKS) :
    (L : PRE --> POST inhibit INH in INTERVAL) ; NET)
   =>
    tickOk : ((M - PRE) + POST) :  updateClocks(.., M - PRE, ....) :
    (L : PRE --> ... ; NET)
   if (active(...) and (T in INTERVAL)) = true .


eq updateClocks((L' -> R1) ; CLOCKS, INTERM-M, (L' : PRE --> ...); NET)
 = (L ->  PRE <= INTERM-M ? R1 : 0/1) ; updateClocks(...) .
```

# 1-Unsafe Nets Reachable?

```
smt-search   tickOk :  m : initClocks(net) : net
       =>*   TICK   : M   : CLOCKS          : NET
  such that (a:Real >= 0/1 and not k-safe(1, M)) = true .
```

Unsafe net reachable if $a \geq 4$

# Problems – And Solutions

- Subsumption $T \sqsubseteq U$ not sufficient
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    - $T \preceq U$ iff $[\![T]\!] \subseteq [\![U]\!]$ for PITPNs

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**Theorem**

*Reachability with $\preceq$-folding terminates when symbolic state space finite!*

# Reachability Analyses with New Folding

1. Maude search: carry symbolic states visited in current behavior
   - stop when current state subsumed by previous symbolic state in same branch
2. Store all visited symbolic states + implement search analysis
   - encode breadth-first search at Maude meta-level

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**Example**

```
search init(net, m0, a < 4 ...)
        =>*   S : φ' || ( TICK : M : CLOCKS : NET )
                such that smtCheck(φ' and not k-safe(1,M)) .
```

returns `No solution`

# EF-Synthesis

For what parameter values is the net not 1-safe?

```
search  [1] init(net, m0, ϕ)
      =>*  S : PHI' ||  ( TICK : M : CLOCKS : NET )
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```

- quantifier elimination of resulting constraint gives desired values of parameters
- $a \geq 4$

# Initial Marking Synthesis

$m_s$ parametric initial marking; $0 \leq x_i \leq 1$ tokens in place $p_i$

```
safety-syn(net, m_s, φ_0, k-safe(1,M))
```

- net 1-safe when $x_1 = x_3 = 0$ and $0 \leq x_2 \leq 1$

# More Analysis

- Analysis with user-defined strategies
- Non-nested temporal properties
- ...

# Comparison with Roméo

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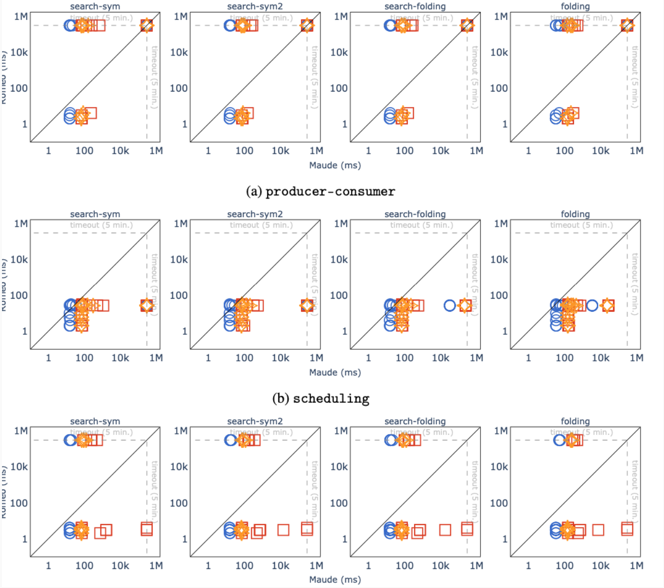
- More:
    - parametric/synthesize initial markings
    - user-defined strategies (restricting possible behaviors)
    - ...

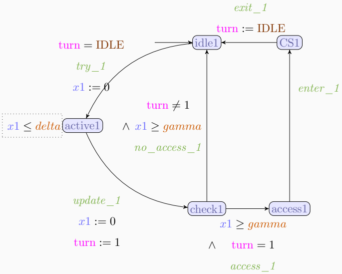
# Benchmarking

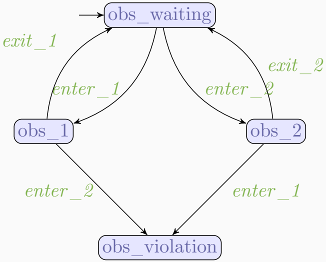
Comparison with Roméo; different implementations and solvers



# Networks of Parametric Timed Automata with Variables

# Networks of Parametric Timed Automata with Variables (NPTAVs)




## "Interpreter" for NPTAVs

- sound/correct
- folding-based analysis
- terminates whenever PZG finite

# Parameter Synthesis: Fischer

**Example**

```
Maude> red synthesis-folding  in 'FISCHER :
             init => (observer @ obs-violation) .
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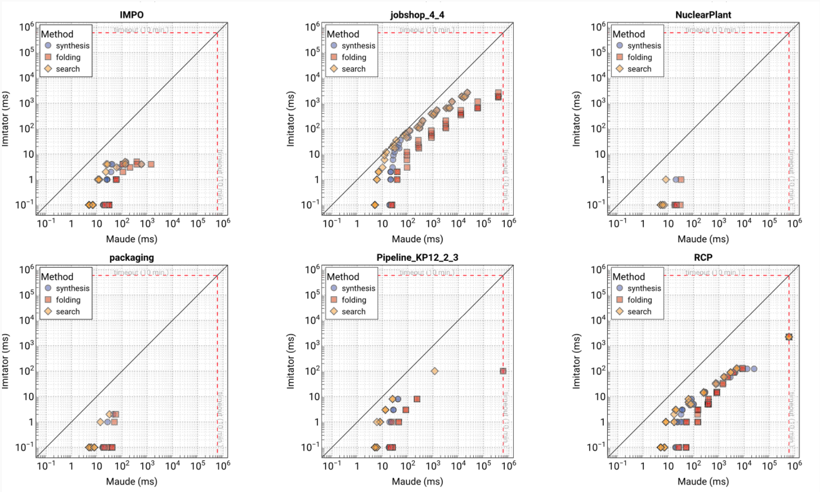
Maude> red synthesis-folding  in 'FISCHER :
              init => (observer @ obs-violation) .

result ConjRelLinRExpr: rr(var(gamma)) + -1 * rr(var(delta)) <= 0
       and -1 * rr(var(delta)) <= 0 and  -1 * rr(var(gamma)) <= 0
```

Bad states reachable when $\gamma \leq \delta$

# Benchmarking: Imitator vs Maude-SMT



# Concluding Remarks

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- Semantics/interpreter for NPTAVs and PITPNs
- Symbolic states (Maude-with-SMT analysis)
- New folding: termination when Imitator/Roméo terminates
- Reachability, parameter synthesis, ...

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  - analysis with strategies
  - parametric markings
  - LTL model checking
  - clocks and parameters in "propositions"

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- Benchmarking
  - worse than Imitator
  - comparable with Roméo (?)

# Concluding Remarks II

- Methods towards "Symbolic Real-Time Maude"
- Maude-with-SMT for not-entirely-trivial systems
- Different subsumption relations; different reachability analysis methods; Fourier-Motzkin quantifier elimination; parameter synthesis; ...
- Extend to classes of systems beyond automata and Petri nets
  - linear arithmetic/reals?
  - topmost rules?
  - ...
- Symbolic full (timed) temporal logic model checking