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Minimisation of Event Structures

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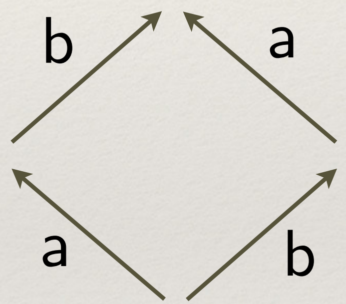
What are we talking about?

True Concurrency

- **Interleaving**
  concurrency reduced to the nondeterministic choice between the possible interleaving

$$a \parallel b \neq ab + ba$$



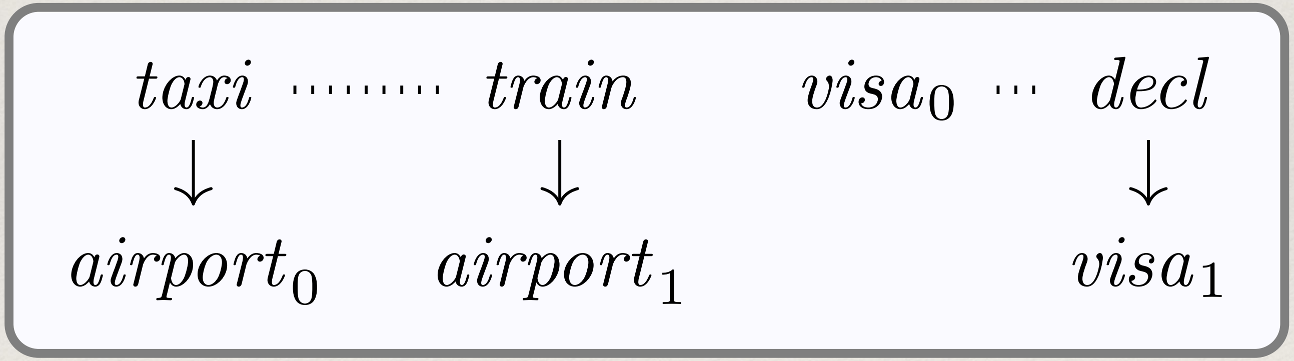
True concurrent semantics

different
concurrency
causality

(Prime) Event structures

- **Prime** event structures [Nielsen,Plotkin, Winskel]
  - E events
  - ≤ causality
  - # conflict
  - λ labelling

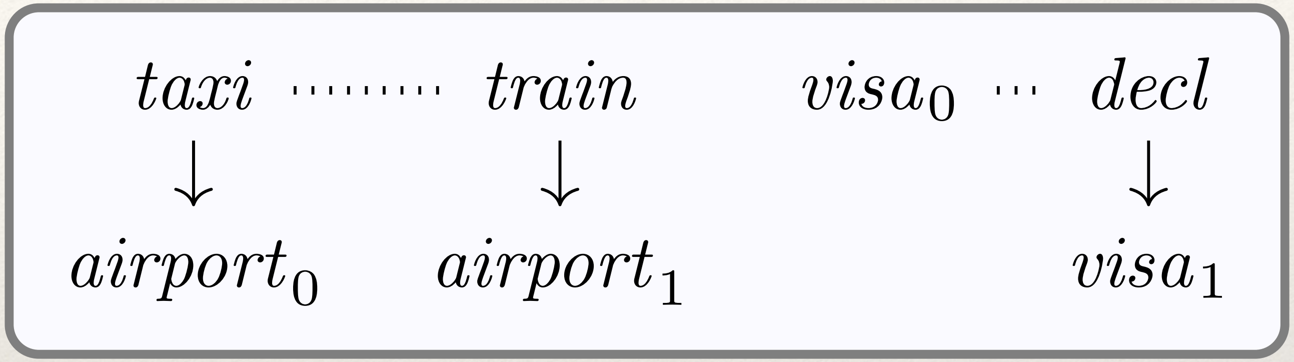
$$(E, \leq, \#, \lambda)$$



Computations as Configurations

(E, ≤, #, λ)

Computations as **configurations**

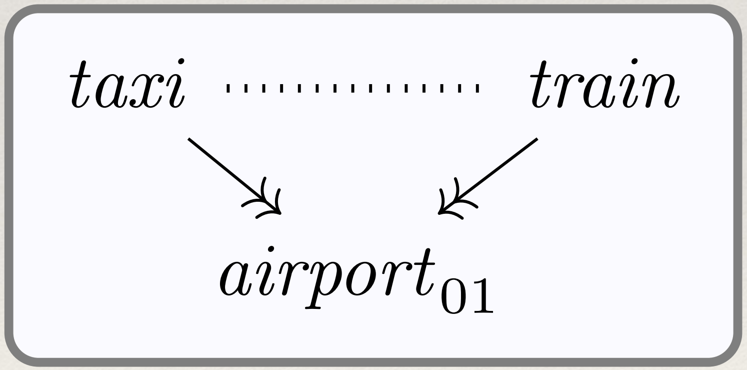


(causally closed, conflict-free)

$$\emptyset \xrightarrow{taxi} \{taxi\} \xrightarrow{visa_0} \{taxi, visa_0\} \xrightarrow{airport_0} \{taxi, visa_0, airport_0\}$$

More expressive ES models

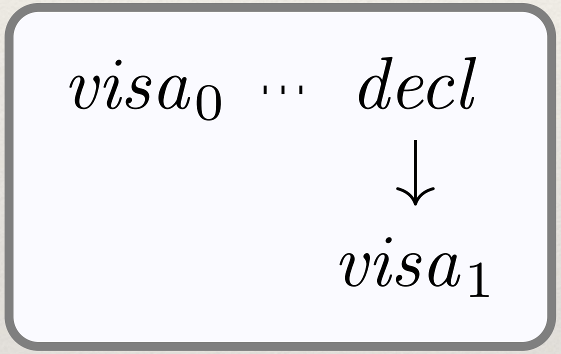
- **Flow event structures** [Boudol, Castellani]
  - Causality replaced by **flow relation** which permits conflictual disjunctive causes

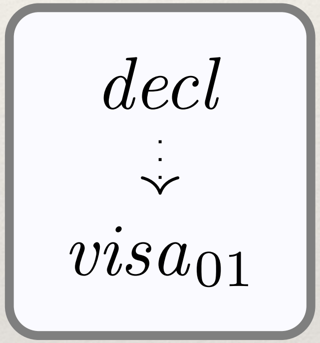


More expressive ES models

- **Asymmetric event structures**

  [Langerak], [B.. Corradini, Montanari]



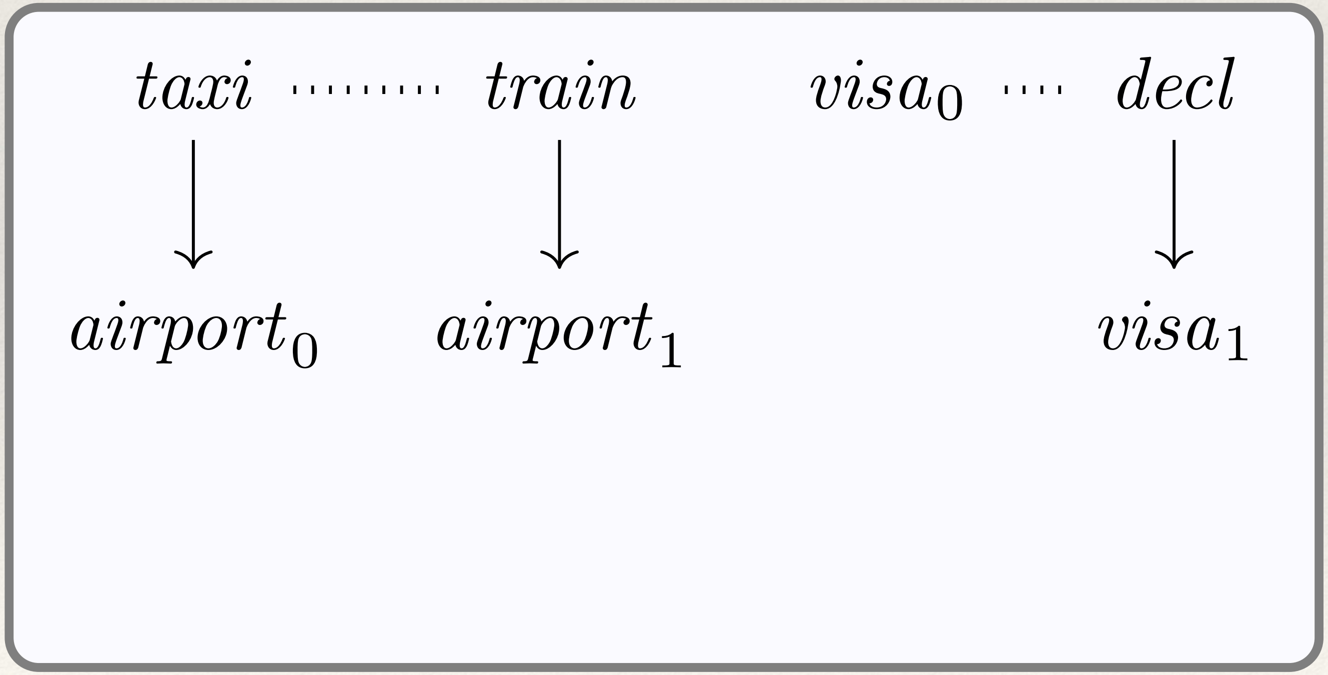


- *Asymmetric conflict*:
  decl can be executed only before $visa_{01}$

Mining processes as ESs

- Processes mined in the form of event structures

[Dumas, García-Bañuelos,Armas-Cervantes, ...]



Original questions

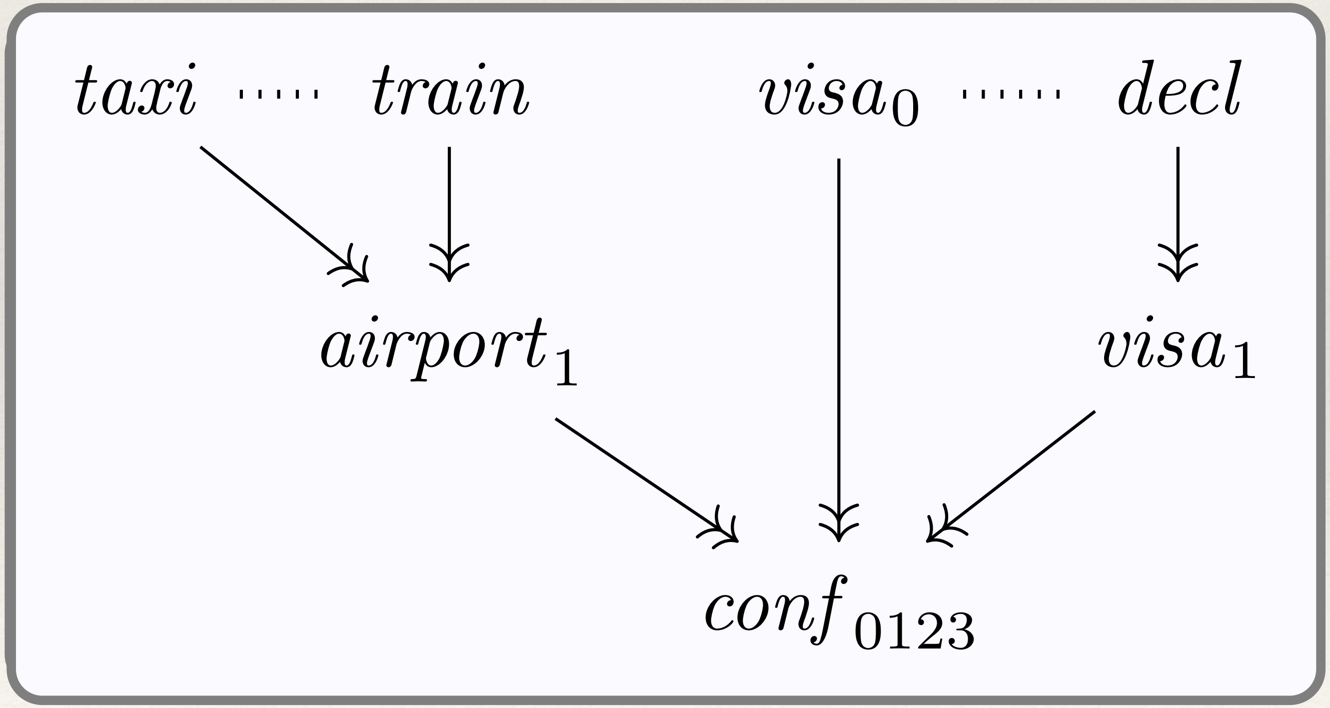
- Can we get **smaller models**, by some form of **quotient**, keeping the **concurrent behaviour unchanged**?

- **Folding**: Surjective $f : E \to E'$, that **merges some events** establishing a **concurrent bisimulation**

- Some work ad hoc in [B., García-Bañuelos,Armas-Cervantes,]

Original questions

- It worked ...



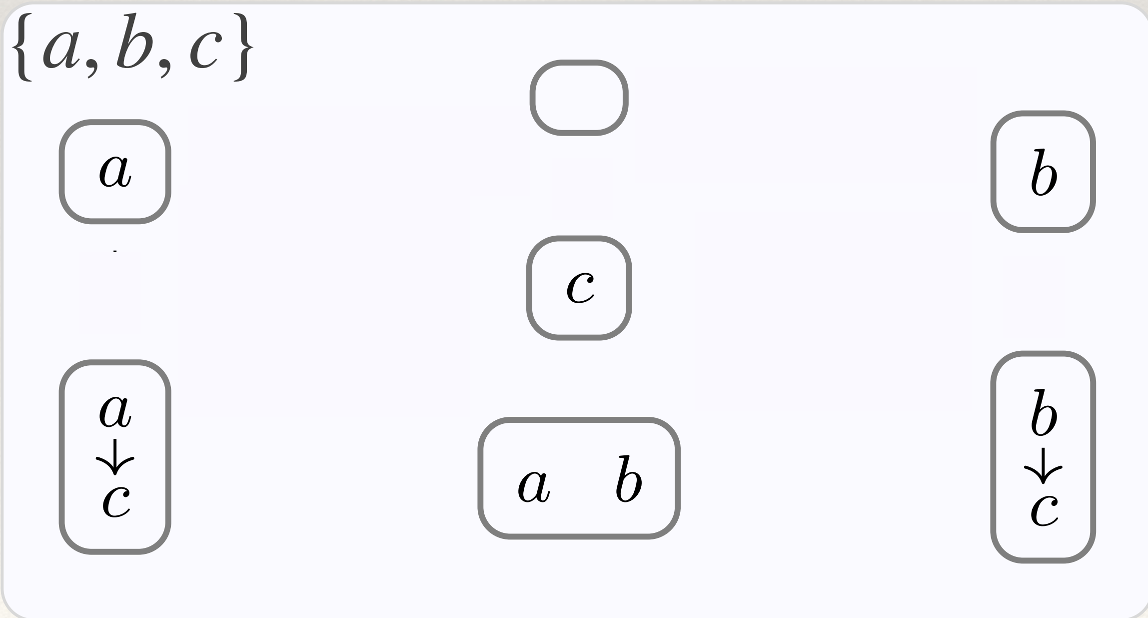
General theory?

General **theory of behaviour-preserving quotients** for **event structures?**

- Is the **notion of folding adequate** for expressing behaviour-preserving quotients?
- Is there a **minimal quotient** in some general class of event structures? Does it exist in **specific subclasses**?
- Can we have a **characterisation** of foldings directly on the ES?

Poset ES

- $\langle E, Conf(E), \lambda \rangle$ [Rensink]
  - E events
  - $Conf(E)$ is a set of **configurations** (posets of events)
  - **Prefix order** on configurations



Stable ES as Poset ES

- Common **stable ES models** are **instances of** poset ES
  - prime ES [Nielsen, Plotkin, Winskel]
  - flow ES [Boudol, Castellani]
  - bundle ES [Langerak]
  - asymmetric ES. [B., Corradini, Montanari]
  - ...

Behavioural Equiv.: Hhp-bisimilarity

fully observes the interplay between
causality, concurrency, non-determinism

HHP-bisimilarity between E and E'

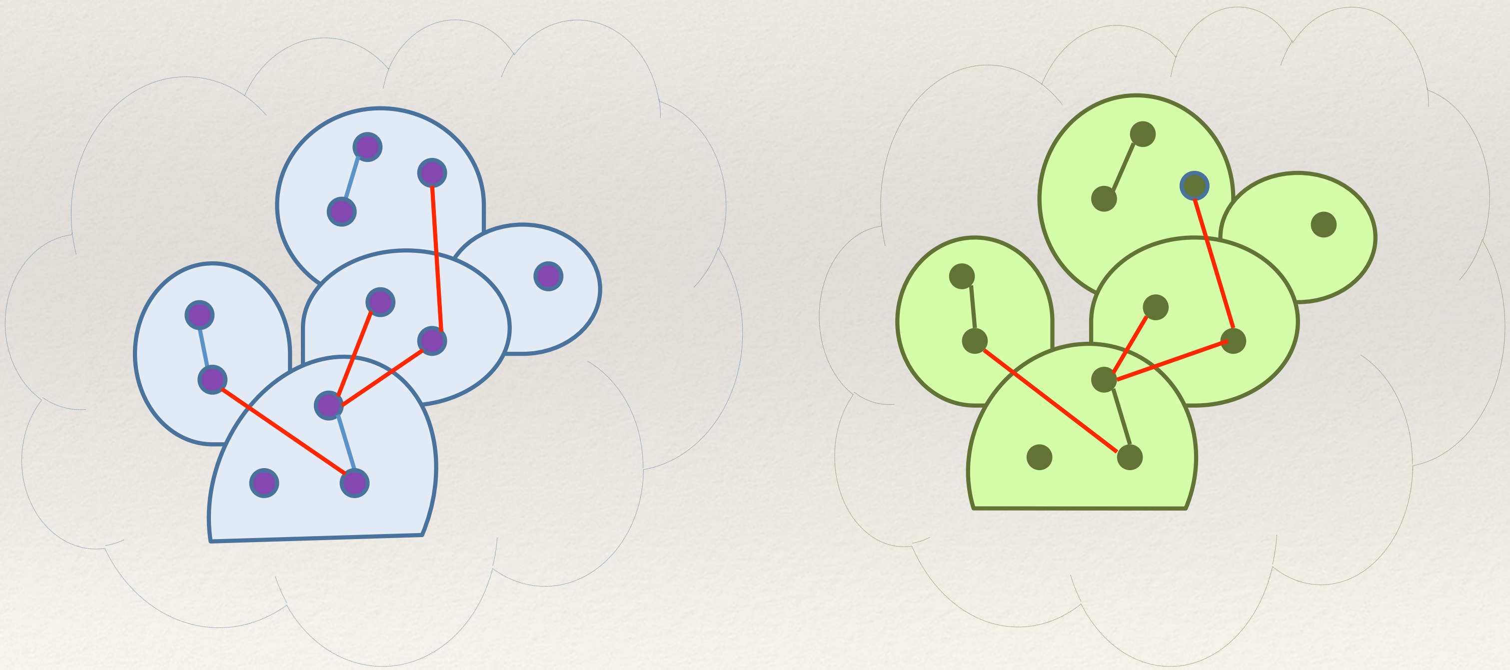
$$R = \{(C, f, C') \mid C \in Conf(\mathsf{E}),\ C' \in Conf(\mathsf{E'}),\ f : C \xrightarrow{\sim} C'\}$$

- Simulation:
  if $(C_1, f, c'_1) \in R$ and $C_1 \xrightarrow{x} C_2$ then $C'_1 \xrightarrow{x'} C'_2$ with $(C_2, f[x \mapsto x'], C'_2) \in R$, and vice versa.

- Down closure:
  if $(C_1, f, c'_1) \in R$ and $C_2 \sqsubseteq C_1$ then $(C_2, f_{|C_2}, f(C_2)) \in R$.

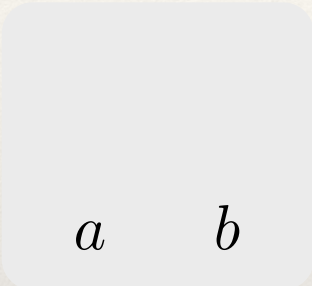
HHP-Bisimilarity

- An event of a system must be simulated by an event of the other with the same history (causal links)



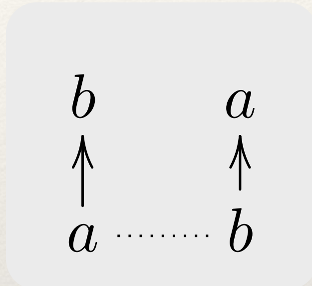
Example

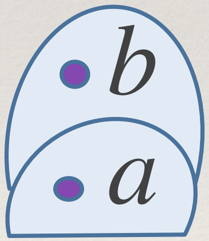
E1: $a \parallel b$

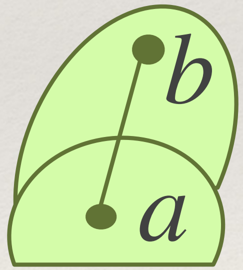


$\not\sim_{hhp}$

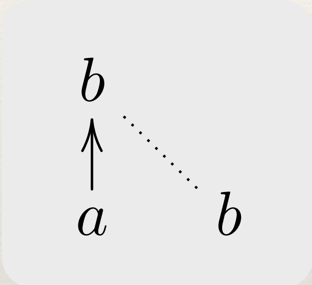
E2: $ab + ba$





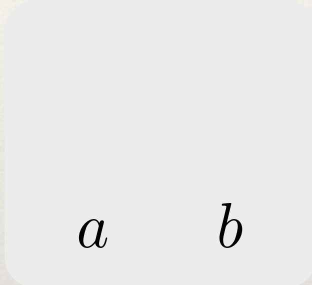


E_1



$$\not\sim_{hhp}$$

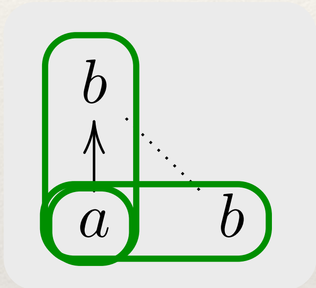
E_2



$$\emptyset \xrightarrow{\{a\}} \{a\} \xrightarrow{\{b\}} \{a < b\}$$

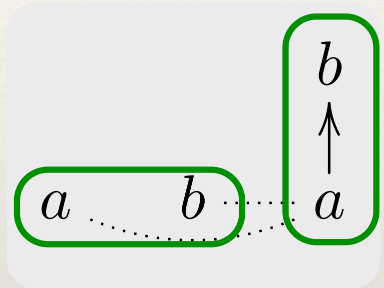
E_1 can perform b **causally dependent** on a while E_2 cannot

E_1



$$\not\sim_{hhp}$$

E_2



$$\emptyset \xrightarrow{\{a\}} \{a\} \xrightarrow{\{b\}} \{a < b\}$$

$$\{a\} \xrightarrow{\{b\}} \{a\ b\}$$

after a, **E_1** can choose between a **causally dependent** and a **concurrent** b while **E_2** cannot

Folding

Morphism

$f : E_1 \to E_2$ function on events, transforms confs into confs

ES

Folding

ES$_f$

Surjective morphism $E \to E'$ such th

$$R_f = \{(C, f_{|C}, f(C)) \mid C$$

is hhp-bisimilarity

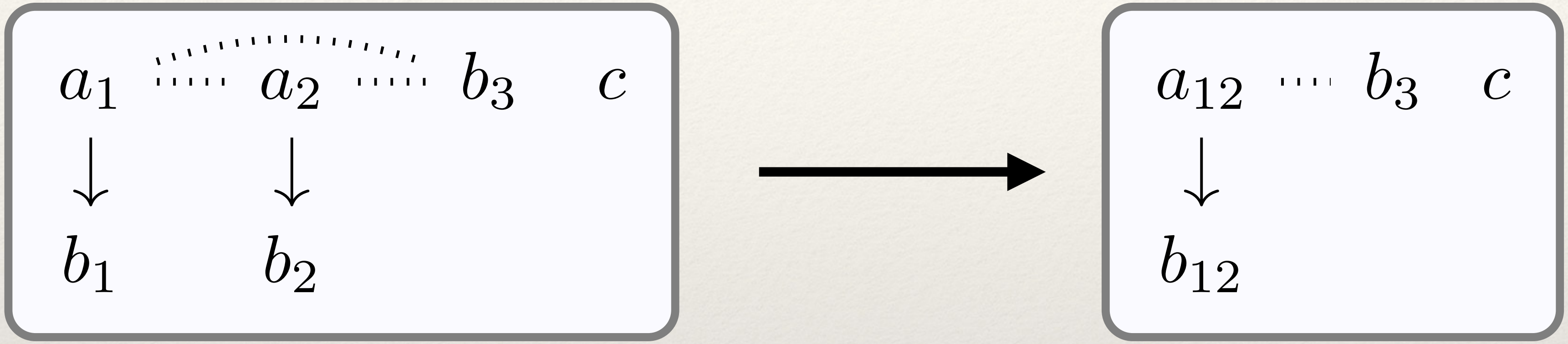
Int.: Merge events without changing the concurrent behaviour

Folding Equivalence $\equiv_f$

def. by $e_1 \equiv_f e_2$ iff $f(e_1) = f(e_2)$

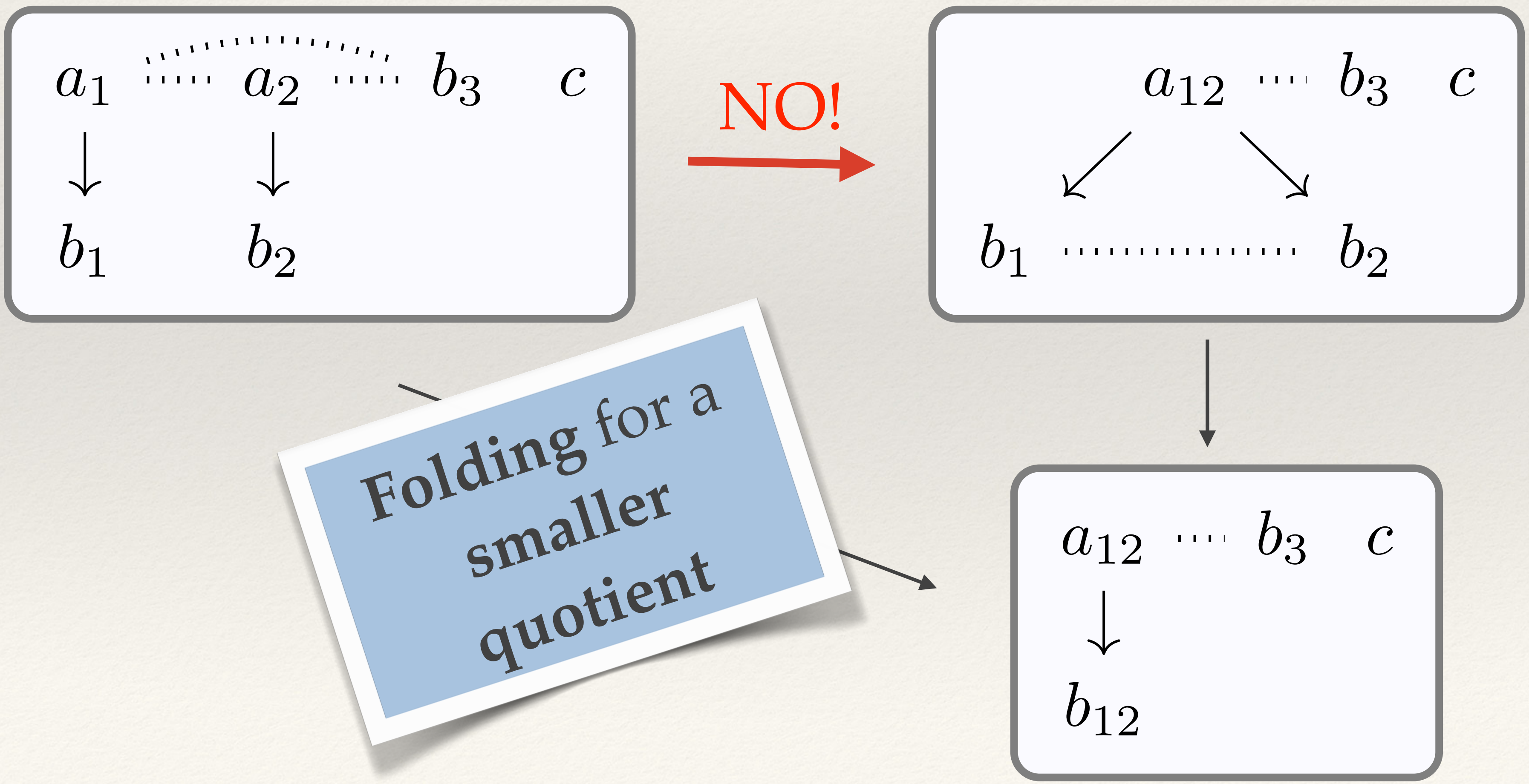
then $E \sim_{hhp} E_{/\equiv_f}$

Folding: Example



Is the notion of folding adequate?

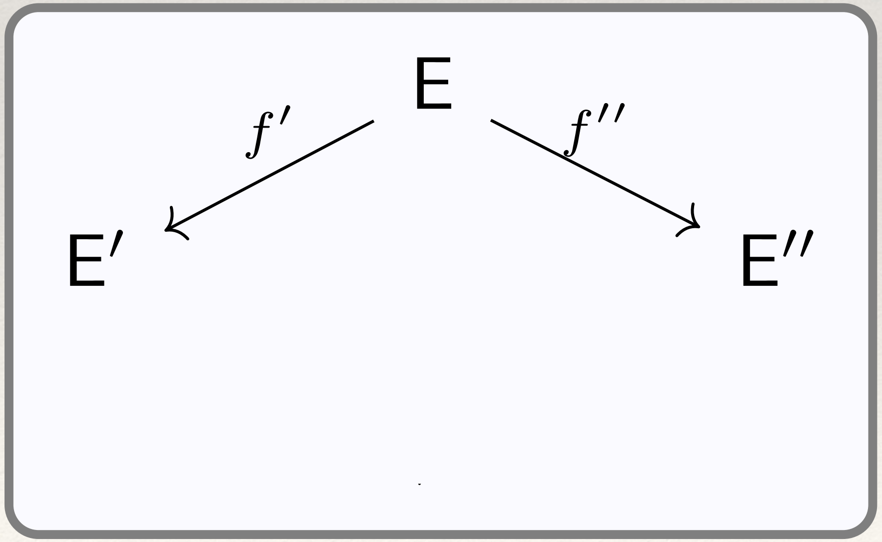
- Not all quotients which preserve behaviour are foldings



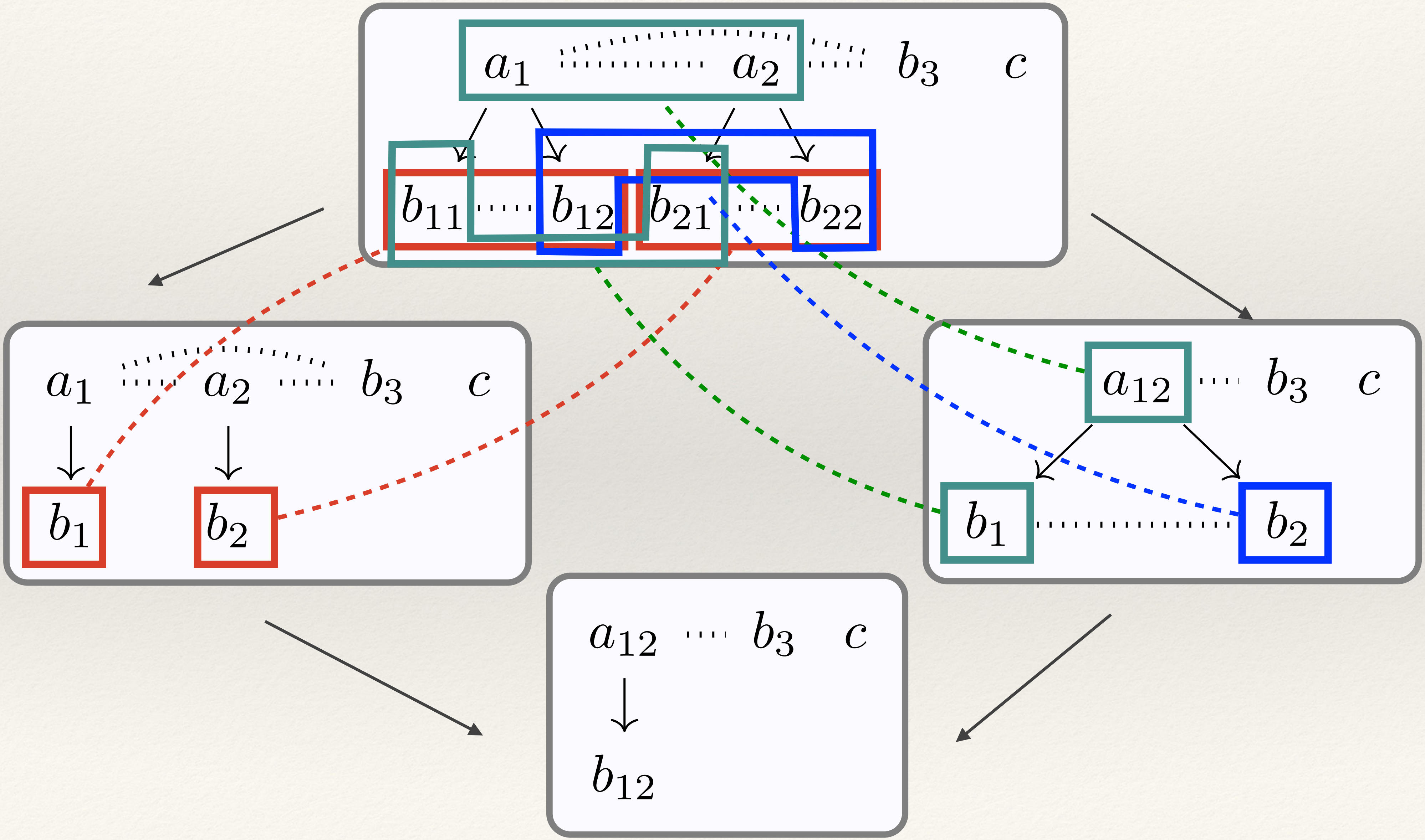
Minimal quotients?

Joining foldings

Joining Foldings
Given two foldings $f : E \to E'$ and $f : E \to E''$ there are always foldings $f : E' \to E_{/\equiv}$ and $f : E'' \to E_{/\equiv}$ where $\equiv$ is $(\equiv_f \cup \equiv_g)^*$



Joining foldings: example



Maximally folded ES / Minimal quotient

Lattice of folding equivalences
Given a poset ES the **folding equivalences** form a complete sublattice of the lattice of equivalences over E

Corollary
Given a poset ES there exists its maximally folded version $\min(E)$

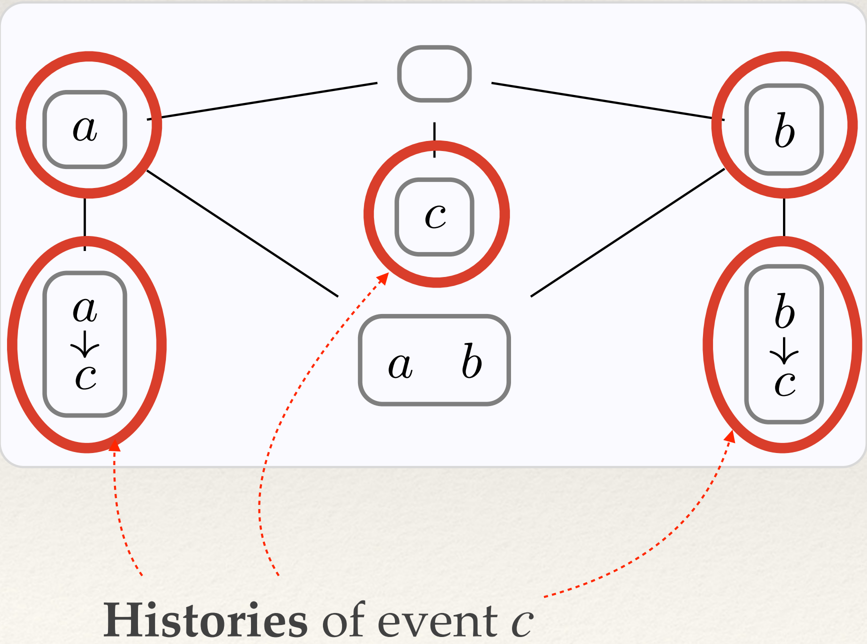
$$\begin{array}{c} \mathsf{E} \\ {\scriptstyle !}\big\downarrow \\ \min(\mathsf{E}) \end{array}$$

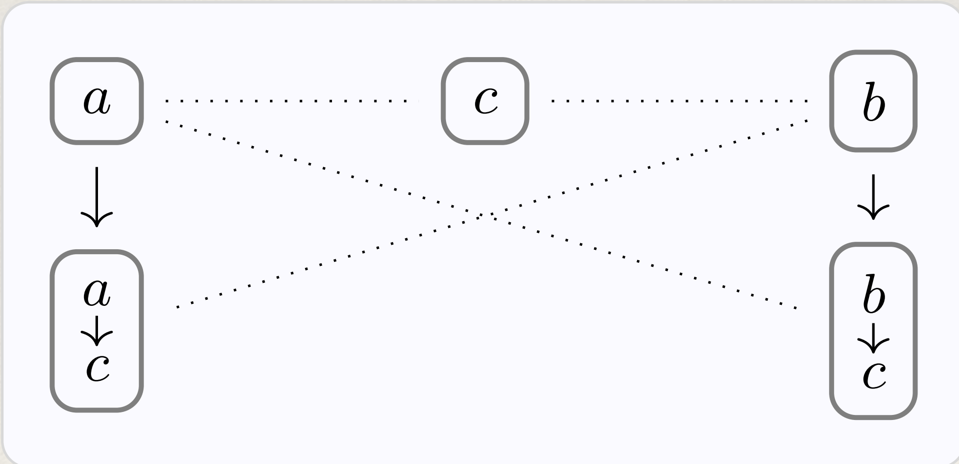
Characterising foldings

Folding through PESs

Theorem: Each poset ES E is the folding of a canonical **Prime** ES

$$\phi_E : \mathbb{P}(E) \to E$$





Folding factorises through PESs

Theorem

Given poset ES E and E' and a morphism $f : E \to E'$ consider

$$\mathsf{E} \xrightarrow{f} \mathsf{E}'$$

f is a folding iff $\mathbb{P}(f)$ is a folding

Foldings on PES

Labels & causes

Conflicts

Concurrency

Characterisation

$f : \mathsf{P} \to \mathsf{P}'$ surjective PES ... phism

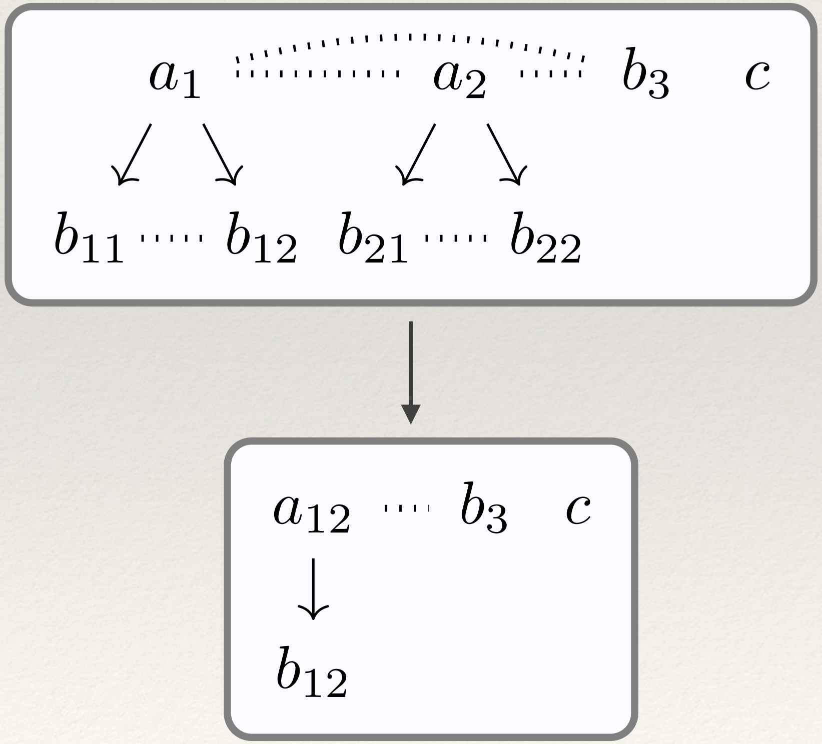
f is a folding iff $\forall\, W \subseteq \mathsf{P}$ ... $\in \mathsf{P}'$

1. if $x \#^{\forall} f^{-1}($ ... $f(x) \# y'$;
2. if $f(x) = f(y)$, $\frown W$ ... $w \frown^{\exists} \{x, y\}$ then $\exists\ z \in \mathsf{P}$ such ... $f(z) = f(x)$ and $\frown(W \cup \{z\})$.

Minimal Foldings for PESs?

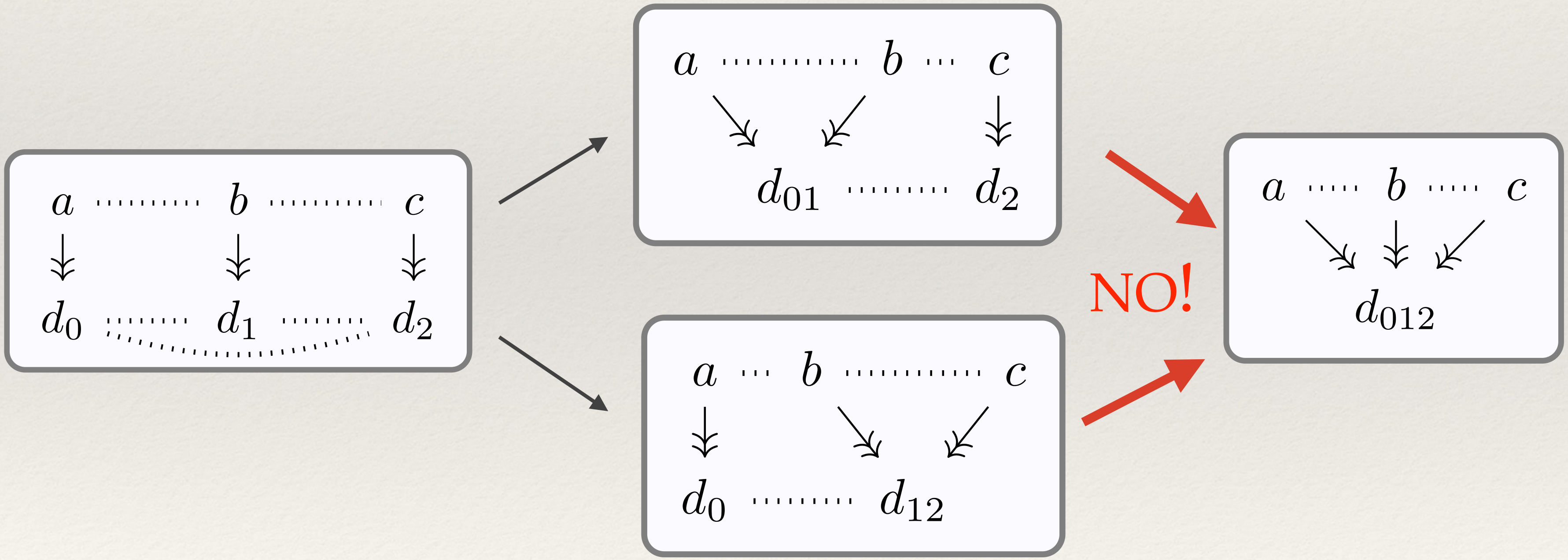
Theorem:
Any PES has maximally folded version in the class of PESs

Example



Minimal Foldings for other classes?

Not true in the class of FESs (and other subclasses)



Conclusion

[Joyal, Nielsen, Winskel]

Foldings as **open maps** (with poset as cat of obs)
-> more abstract and general view of the results?

Folding algorithms (for PESs and other subclasses)?

Folding for operational models (finitary representations of ESs, e.g. Petri nets)