Logical Foundations of Cyber-Physical Systems

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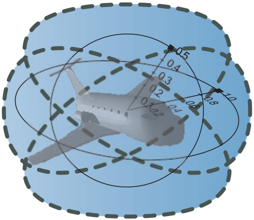


Cyber-Physical Systems Analysis: Aircraft Example

Which control decisions are safe for aircraft collision avoidance?

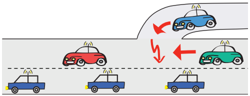
Cyber-Physical Systems

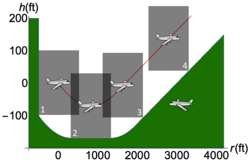
CPSs combine cyber capabilities with physical capabilities
to solve problems that neither part could solve alone.

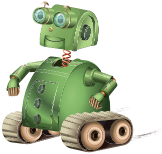
Cyber-Physical Systems Promise Transformative Impact!

Prospects: Safety & Efficiency

(Autonomous) cars | (Auto)Pilot support | Robots near humans







Cyber-Physical Systems

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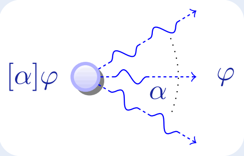
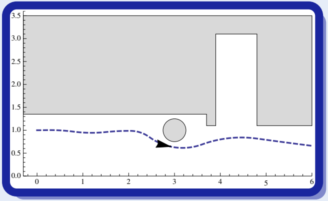
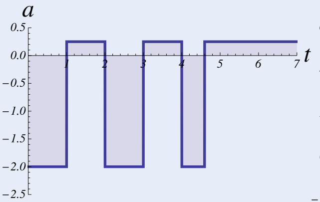
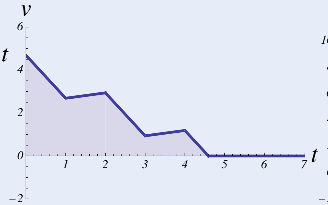
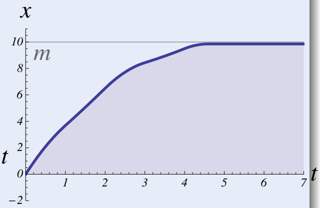
Concept (Differential Dynamic Logic) (JAR'08,LICS'12)





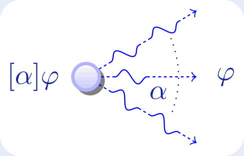


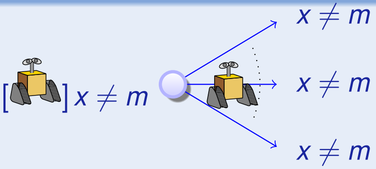


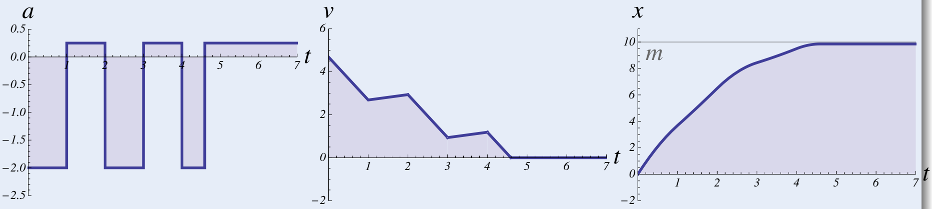


CPS Analysis

Concept (Differential Dynamic Logic) (JAR'08,LICS'12)







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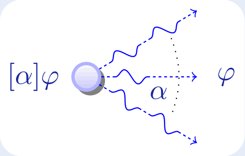
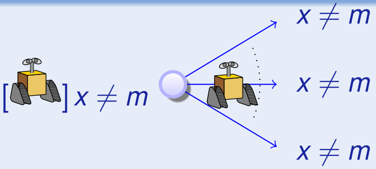
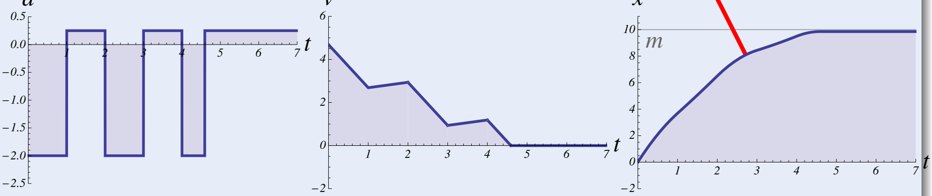
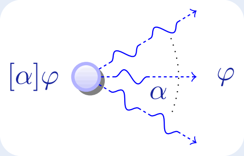




$$x' = v, v' = a$$



Concept (Differential Dynamic Logic) (JAR'08,LICS'12)



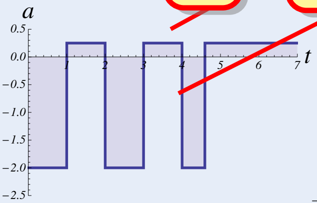
seq. compose

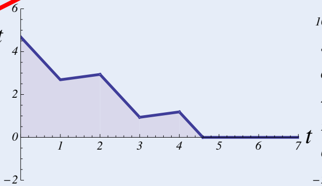
$(\texttt{if}(SB(x,m))\, a := -b)\ ;\ x' = v, v' = a$

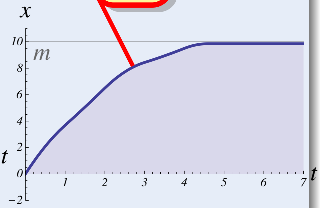
cond

assign

ODE







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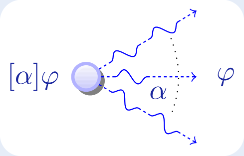


seq. compose

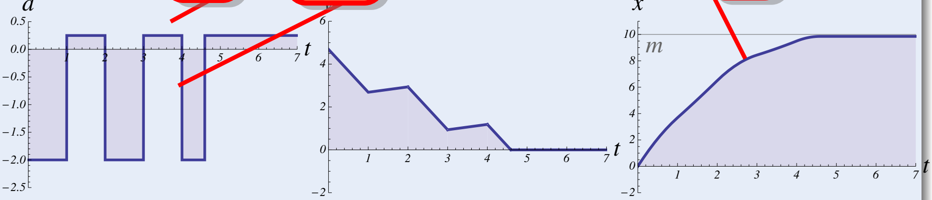
nondet. repeat

$$((\mathtt{if}(SB(x,m))\; a := -b) \; ; \; x' = v, v' = a)^*$$

cond

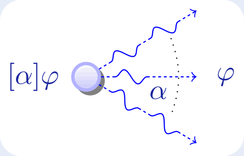
assign

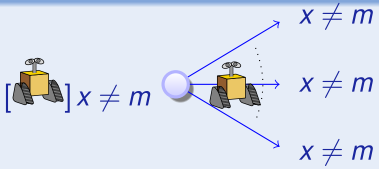
ODE



CPS Analysis

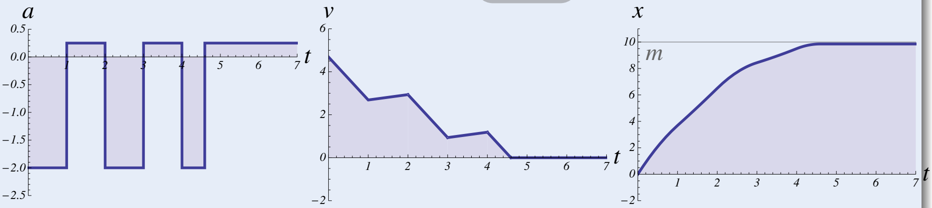
Concept (Differential Dynamic Logic) (JAR'08,LICS'12)





$$[((\mathtt{if}(SB(x,m))\ a := -b) ;\ x' = v, v' = a)^*]\underbrace{x \neq m}_{\text{post}}$$

all runs



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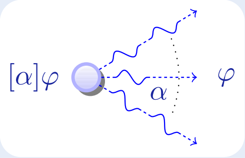
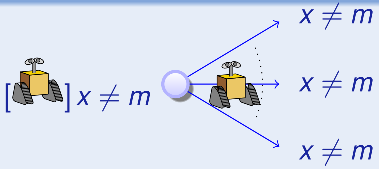
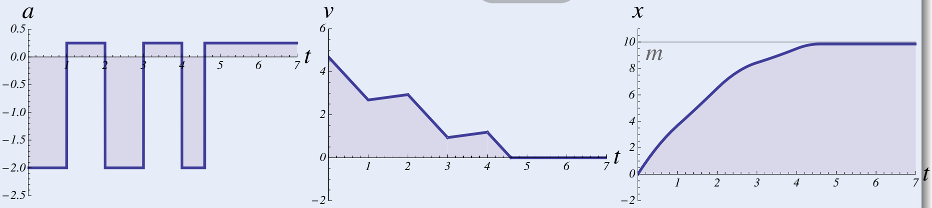




$$\underbrace{x \neq m \wedge b > 0}_{\text{init}} \rightarrow \big[\big((\mathtt{if}(\mathrm{SB}(x,m))\, a := -b)\,;\ x' = v, v' = a\big)^*\big]\underbrace{x \neq m}_{\text{post}}$$

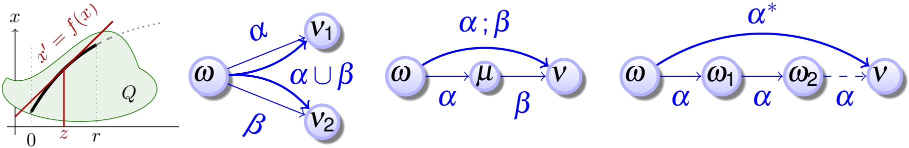
all runs



Differential Dynamic Logic dL

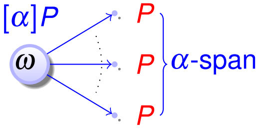
Definition (Hybrid program)

$$\alpha, \beta \ ::= \ x := e \mid ?Q \mid x' = f(x) \,\&\, Q \mid \alpha \cup \beta \mid \alpha ; \beta \mid \alpha^*$$



Definition (Differential dynamic logic)

$$P, Q ::= e \geq \tilde{e} \mid \neg P \mid P \wedge Q \mid P \vee Q \mid P \rightarrow Q \mid \forall x\, P \mid \exists x\, P \mid [\alpha]P \mid \langle \alpha \rangle P$$



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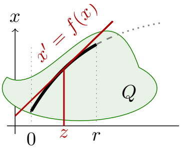

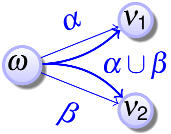

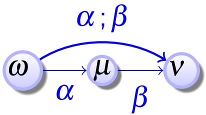

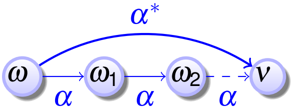

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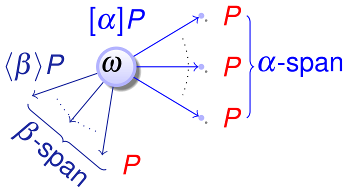

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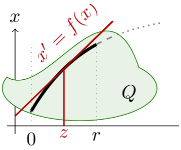

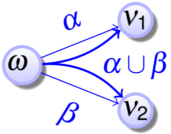

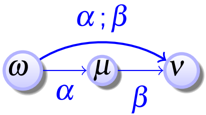

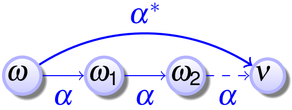

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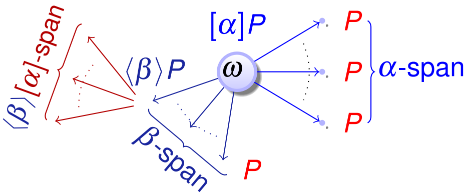


Differential Dynamic Logic dL: Axiomatization

equations of truth

[:=] $[x := e]P(x) \leftrightarrow P(e)$

[?] $[?Q]P \leftrightarrow (Q \rightarrow P)$

[′] $[x' = f(x)]P \leftrightarrow \forall t{\geq}0\,[x := x(t)]P$ $\quad (x'(t) = f(x))$

[∪] $[\alpha \cup \beta]P \leftrightarrow [\alpha]P \wedge [\beta]P$

[;] $[\alpha;\beta]P \leftrightarrow [\alpha][\beta]P$

[*] $[\alpha^*]P \leftrightarrow P \wedge [\alpha][\alpha^*]P$

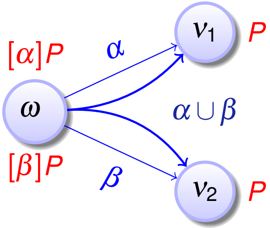
K $[\alpha](P \rightarrow Q) \rightarrow ([\alpha]P \rightarrow [\alpha]Q)$

I $[\alpha^*]P \leftrightarrow P \wedge [\alpha^*](P \rightarrow [\alpha]P)$

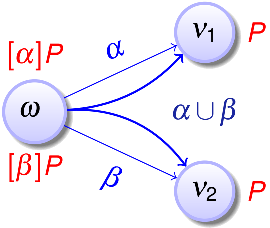
C $[\alpha^*]\forall v{>}0\,(P(v) \rightarrow \langle\alpha\rangle P(v-1)) \rightarrow \forall v\,(P(v) \rightarrow \langle\alpha^*\rangle \exists v{\leq}0\,P(v))$

LICS'12,JAR'17

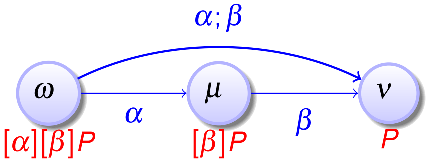
$$[\alpha \cup \beta]P \leftrightarrow [\alpha]P \wedge [\beta]P$$



$$[\alpha \cup \beta]P \leftrightarrow [\alpha]P \land [\beta]P$$

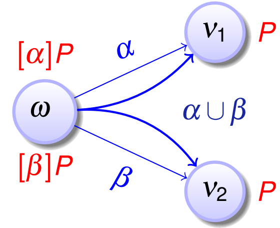


$$[\alpha;\beta]P \leftrightarrow [\alpha][\beta]P$$

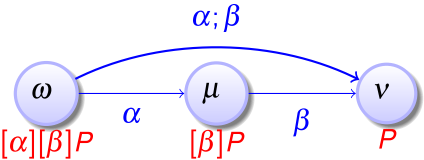


Proofs for CPS

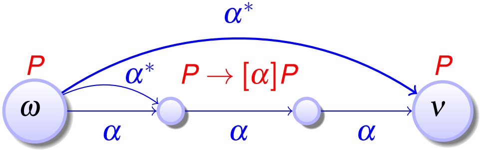
$[\alpha \cup \beta]P \leftrightarrow [\alpha]P \wedge [\beta]P$



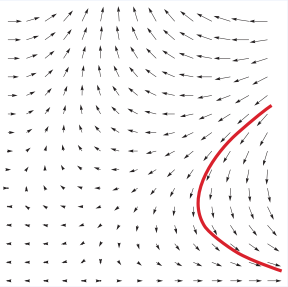
$[\alpha;\beta]P \leftrightarrow [\alpha][\beta]P$



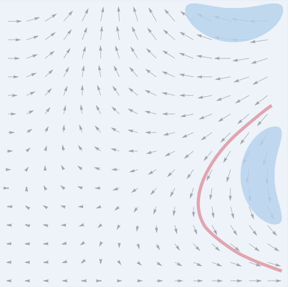
$[\alpha^*]P \leftrightarrow P \wedge [\alpha^*](P \rightarrow [\alpha]P)$



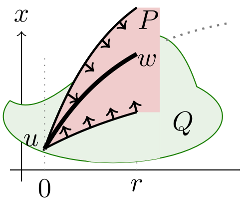
Differential Invariant



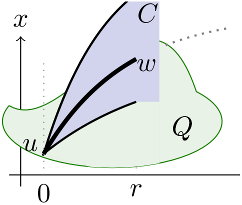
Differential Cut



Differential Ghost



$x' = f(x) \,\&\, Q$



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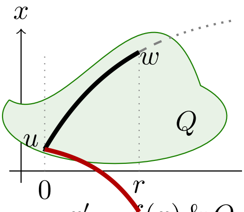
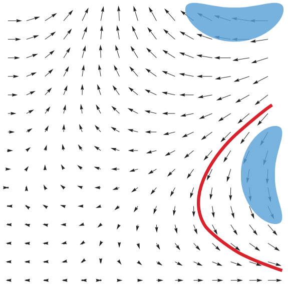
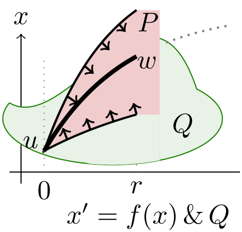


$x' = f(x) \,\&\, Q$

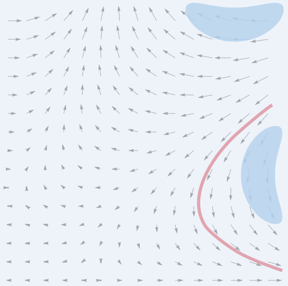
Differential Invariants for Differential Equations

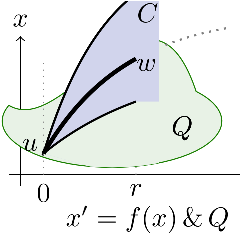
Differential Invariant





Differential Cut





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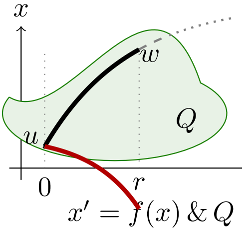

Differential Invariants for Differential Equations

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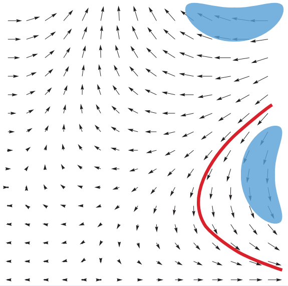

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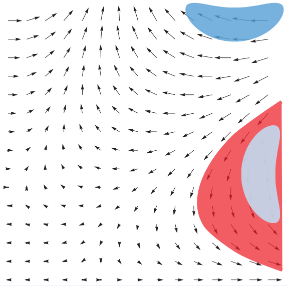

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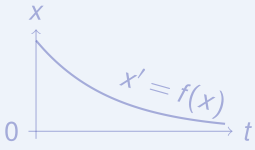
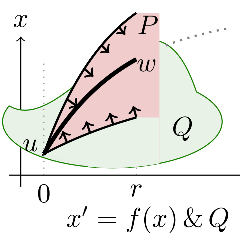
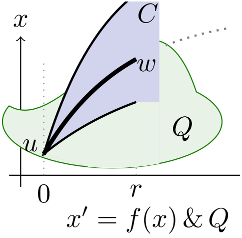
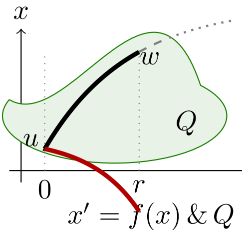


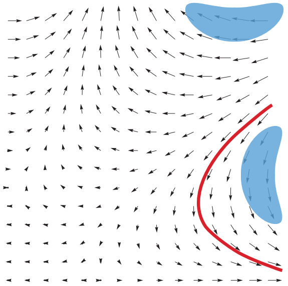






Differential Invariants for Differential Equations

Differential Invariant



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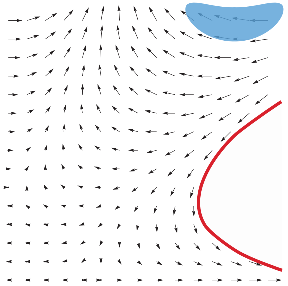
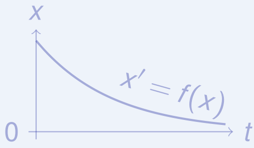
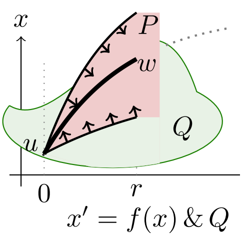
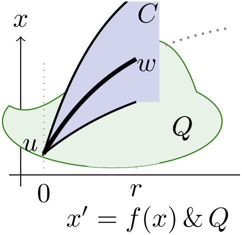
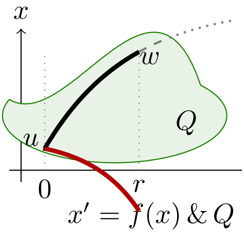


Differential Ghost



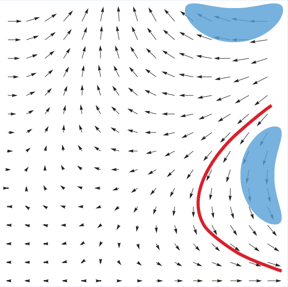


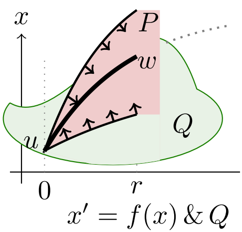




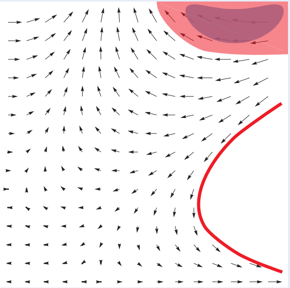
Differential Invariants for Differential Equations

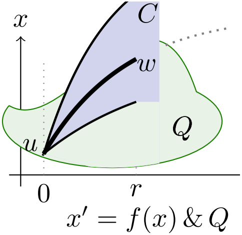
Differential Invariant



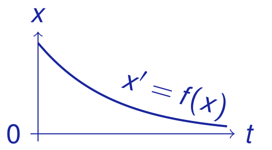


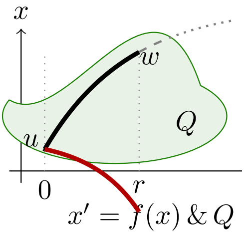
Differential Cut





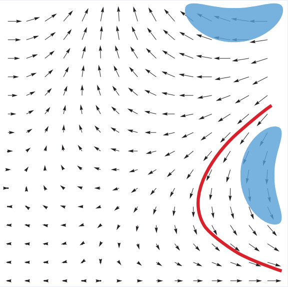
Differential Ghost

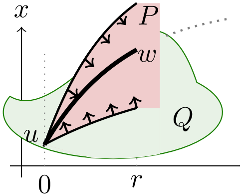




Differential Invariants for Differential Equations

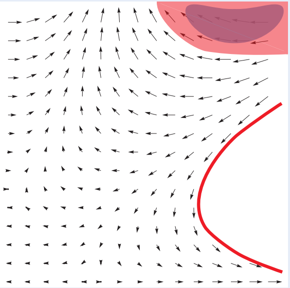
Differential Invariant

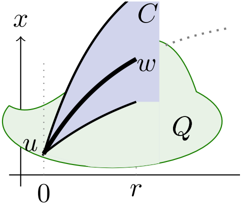




$x' = f(x) \,\&\, Q$

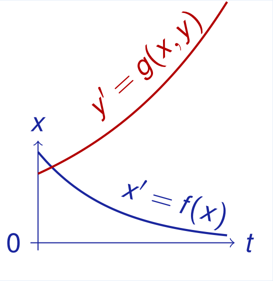
Differential Cut

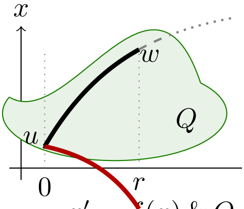




$x' = f(x) \,\&\, Q$

Differential Ghost

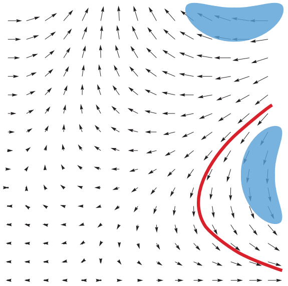




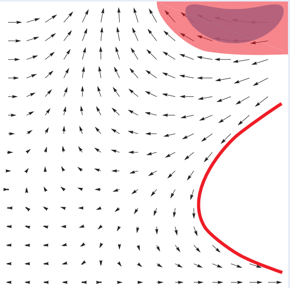
$x' = f(x) \,\&\, Q$

Differential Invariants for Differential Equations

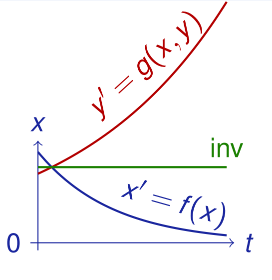
Differential Invariant

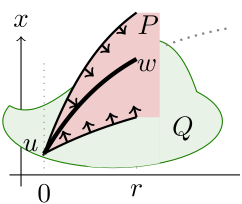


Differential Cut

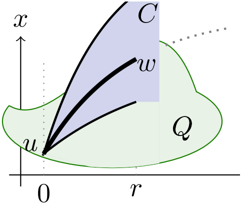


Differential Ghost

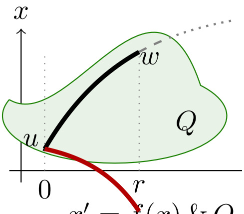




$x' = f(x) \,\&\, Q$



$x' = f(x) \,\&\, Q$



$x' = f(x) \,\&\, Q$

Differential Invariants for Differential Equations

Differential Invariant

$$\frac{Q \to [x' := f(x)](P)'}{P \to [x' = f(x) \,\&\, Q]P}$$

Differential Cut

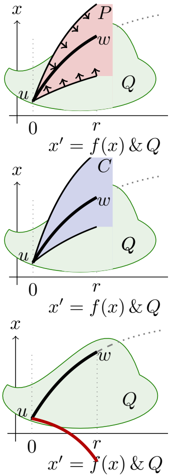
$$\frac{P \to [x' = f(x) \,\&\, Q]C \quad P \to [x' = f(x) \,\&\, Q \land C]P}{P \to [x' = f(x) \,\&\, Q]P}$$

Differential Ghost

$$\frac{P \leftrightarrow \exists y\, G \quad G \to [x' = f(x), y' = g(x,y) \,\&\, Q]G}{P \to [x' = f(x) \,\&\, Q]P}$$

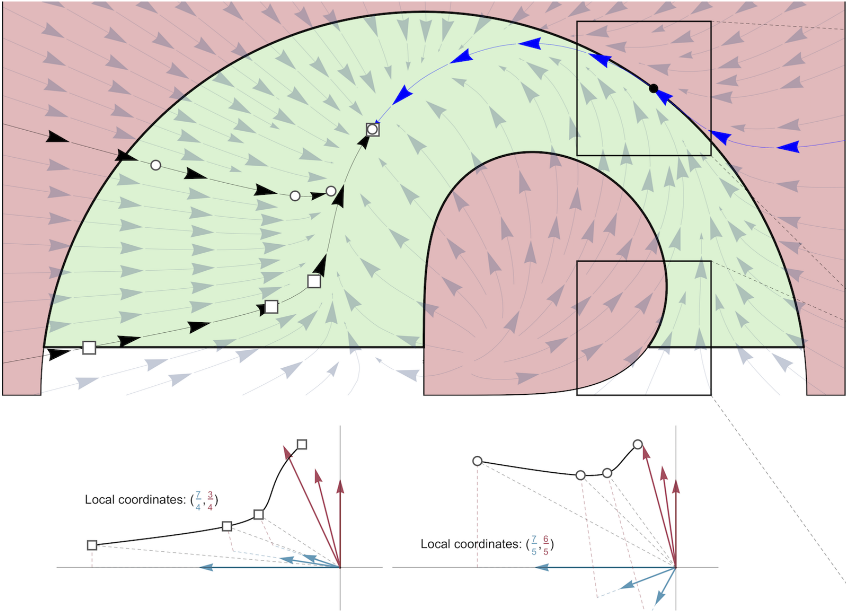
deductive power added DI $\prec$ DI+DC $\prec$ DI+DC+DG

$$[\![(e)']\!]_v = \sum_x v(x') \frac{\partial [\![e]\!]}{\partial x}(v)$$



Springer'10, LMCS'12, LICS'12,JAR'17,LICS'18,JACM'20

Completeness for Differential Equation Invariants



LICS'18,JACM'20

Differential Equation Axiomatization

Theorem (Algebraic Completeness) (LICS'18,JACM'20)

dL *calculus is a sound & complete axiomatization of algebraic invariants of polynomial differential equations. They are decidable by DI,DC,DG in* dL.

Theorem (Semialgebraic Completeness) (LICS'18,JACM'20)

dL *calculus with RI is a sound & complete axiomatization of semialgebraic invariants of differential equations. They are decidable in* dL.

Differential Equation Axiomatization

Theorem (Algebraic Completeness) (LICS'18,JACM'20)

dL *calculus is a sound & complete axiomatization of algebraic invariants of polynomial differential equations. They are decidable with a derived axiom:*

$$\text{(DRI)} \quad [x' = f(x)\,\&\,Q]p = 0 \leftrightarrow \big(Q \rightarrow p^{\dot{}(*)} = 0\big) \qquad (Q \text{ open})$$

Theorem (Semialgebraic Completeness) (LICS'18,JACM'20)

dL *calculus with RI is a sound & complete axiomatization of semialgebraic invariants of differential equations. They are decidable with derived axiom:*

$$\text{(SAI)} \quad \forall x\,(P \rightarrow [x' = f(x)]P) \leftrightarrow \forall x\,\big(P \rightarrow P^{\dot{}(*)}\big) \land \forall x\,\big(\neg P \rightarrow (\neg P)^{\dot{}(-*)}\big)$$

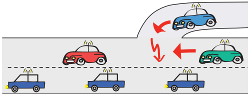
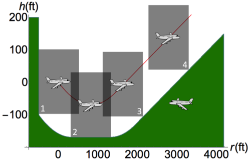
Definable $p^{\dot{}(*)}$ is short for *all/significant* Lie derivative w.r.t. ODE
Definable $p^{\dot{}(-*)}$ is w.r.t. backwards ODE $x' = -f(x)$.

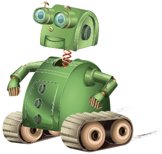
Applications with Impact

Prospects: Safety & Efficiency

(Autonomous) cars | (Auto)Pilot support | Robots near humans



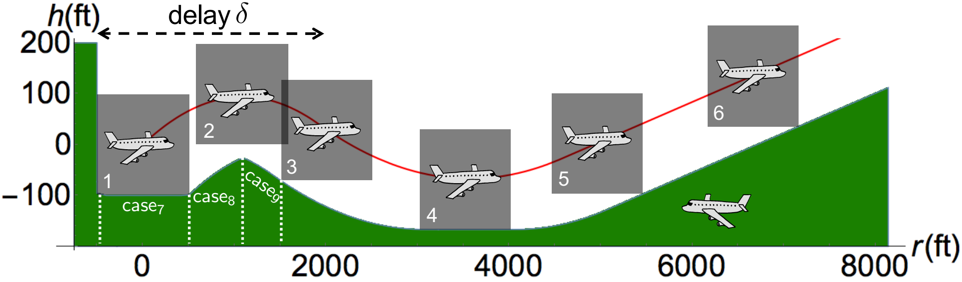





Cyber-Physical Systems

CPSs combine cyber capabilities with physical capabilities
to solve problems that neither part could solve alone.

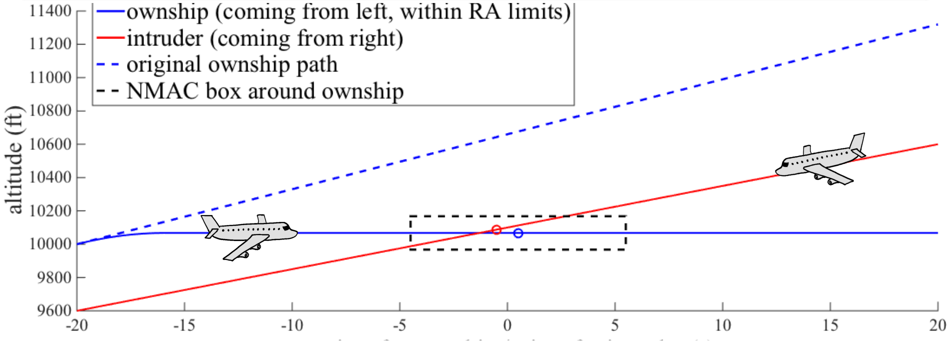
- Developed by the FAA to replace current TCAS in aircraft
- Approximately optimizes Markov Decision Process on a grid
- Advisory from lookup tables with numerous 5D interpolation regions



1. Identified safe region for each advisory symbolically
2. Proved safety for hybrid systems flight model in KeYmaera X

STTT'17

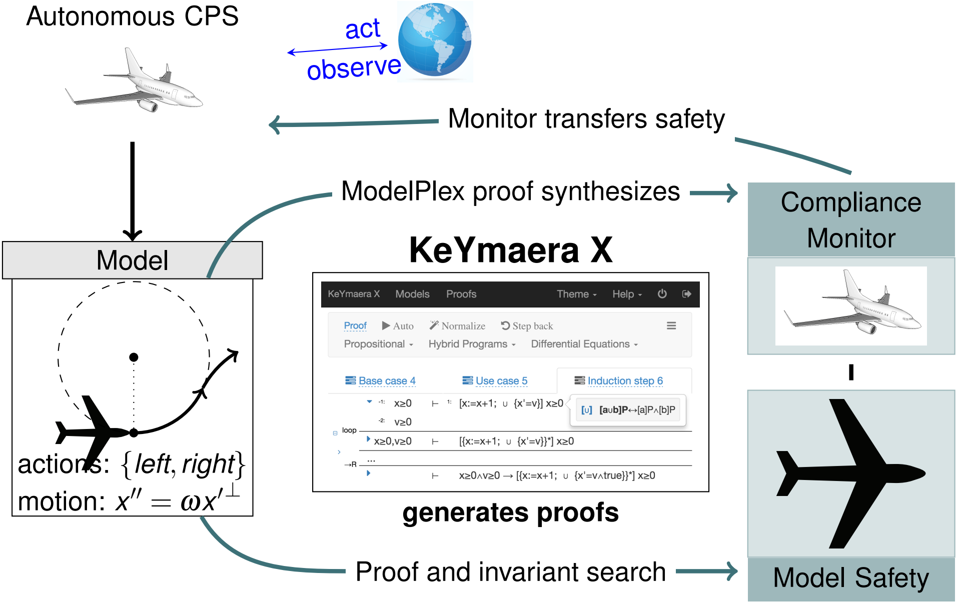
ACAS X table comparison shows safe advisory in 97.7% of the 648,591,384,375 states compared (15,160,434,734 counterexamples).



ACAS X issues DNC advisory, which induces collision unless corrected

STTT'17

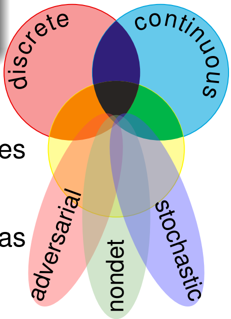
Approach: Safety Proofs for Autonomous CPS

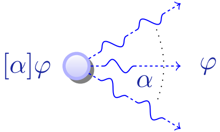


Logical Foundations of Cyber-Physical Systems

differential dynamic logic

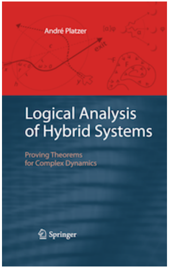
$\text{dL} = \text{DL} + \text{HP}$



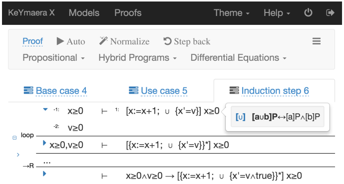


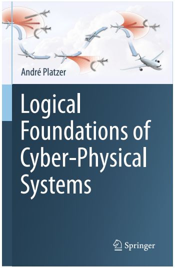
- Strong analytic foundations
- Practical reasoning advances
- Significant applications
- Catalyze many science areas

- Logic & Proofs for CPS
- Programming languages
- Theorem proving
- Multi-dynamical systems



KeYmaera X

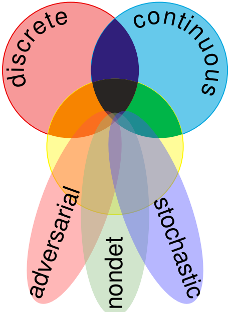




Future CPS Challenges

Numerous wonders remain to be discovered

- Verified CPS implementations by ModelPlex FMSD'16
- Correct CPS execution PLDI'18
- CPS proof and tactic languages+libraries ITP'17
- Big CPS built from safe components STTT'18
- Real arithmetic: Scalable and verified CADE'09
- Scalable stochastic dynamics reasoning CADE'11
- Concurrent CPS
- Invariant generation FM'19
- Safe AI autonomy in CPS AAAI'18
- Correct model transformation FM'14
- Refinement + system property proofs LICS'16
- CPS information flow LICS'18
- Hybrid games TOCL'15



CPSs deserve proofs as safety evidence!

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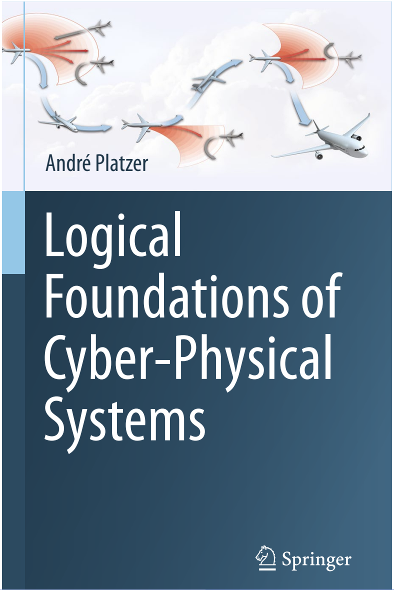

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